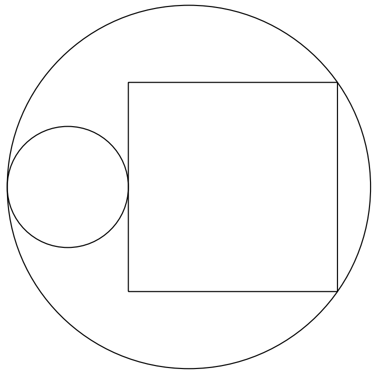
The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

1996 Australian Intermediate Contest Problems

1. Circles of radii 12 and 4 units touch as shown. A square is drawn inside the larger circle touching it and the smaller circle as shown. The point of contact of the smaller circle with the square is the midpoint of one side of the square.



   Find the exact length of the side of the square in the form
   $$\frac{a+\sqrt{b}}{c},$$
   where $a, b, c \in \mathbb{Z}$.

2. *Non-open-box Numbers* are integers greater than 5 which cannot be the surface area of a rectangular box that is open at the top and whose side lengths are integers.
   (a) Find the smallest five Non-open-box Numbers.
   (b) By considering numbers of form $16p + q$ or otherwise, prove that there are exactly two odd Non-open-box Numbers.

3. Let k be a semicircle with diameter AB. Let D be a point such that $AB = AD$ and AD intersects k at E. Let F be the point on the chord AE such that $DE = EF$. Let BF extended meet k at C.

   Show that $\angle BAE = 2\angle EAC$.

4. In a competition, four soccer teams played each other once. The incomplete table below, in which the four teams are listed alphabetically, gives part of the situation when some of the matches have been played. The letters represent digits. Each letter represents the same digit when it appears, and different letters represent different digits. Two points are given for a win, zero points for a loss and one point to each side for a draw.

Teams	Allstars	Bigboots	Canners	Dribblers
No. of matches played	s		p	k
No. of matches won		h	s	
No. of matches drawn	k		k	
No. of matches lost			h	
Total goals for	h	m	t	
Total goals against	p	m		
Total points			m	

   List the matches played and the score for each match.

5. Let $a, b, c \in \mathbb{Z}$ such that $a + b + c = 0$.

   Prove that $2(a^4 + b^4 + c^4)$ is the square of an integer.