The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

1997 Australian Intermediate Contest Problems

1. AB and CD are two chords of a circle, intersecting at X.

   If $AX = AC$, prove that $DX = DB$.

2. On Planet Rhinochromos, 19971997 male monsters are to be married to the same number of female monsters. The same number of male monsters as females have purple noses; the rest have beige noses. The matching of males and females is performed randomly by Rhinochromos government monster psychologists.

   Show that the number of mixed marriages, i.e. marriages of partners with different nose colours is even.

3. The incircle of a right-angled triangle touches the hypotenuse at point P, and P divides the hypotenuse into lengths a and b.

   Show that the area of the triangle is ab.

4. The teacher wrote a positive integer on the board. One student said 'the number is exactly divisible by 2'. A second student said 'it is exactly divisible by 3'. A third student said 'it is exactly divisible by 4', and so on until the thirtieth student said 'it is exactly divisible by 31'. The teacher said that that all the statements were true except two, and the students who made them spoke one after the other.

   Which were the two incorrect statements?

5. The list

   $$\begin{array}{c} 10234567 \\ 10234576 \\ 10234657 \\ \vdots \\ 76543210 \end{array}$$

   contains all the 8-digit numbers which can be made from the digits $0, 1, 2, 3, 4, 5, 6, 7$, used once each, listed from smallest to largest.

   What is the $20\,000^{\text{th}}$ number in the list?