The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

1998 Australian Intermediate Mathematics Olympiad Problems

1. Four dice, coloured red, blue, green and black are rolled.

   In how many ways can the product of the numbers rolled equal 36 (where, for example, a red 4 is considered different to a blue 4)?

2. Find the smallest positive integer by which 139 392 must be multiplied, in order that the answer is a perfect cube.

3. Find the last digit of 1998^{1998}.

4. A square is inscribed in a circle which is inscribed in an equilateral triangle. The side length is 60 cm.

   What is the area of the square in square centimetres?

5. The real numbers x and y satisfy both the equations:

$$x + y + \frac{x}{y} = 19 \quad \text{and} \quad \frac{x(x+y)}{y} = 60.$$

   Find the sum of all possible values for $x + y$.

6. Point P is 60 units from the centre of a circle C, whose radius is 100 units.

   How many chords with integer unit length through P are there?

7. We observe that

$$34\,117 = 166^2 + 81^2 = 159^2 + 94^2$$

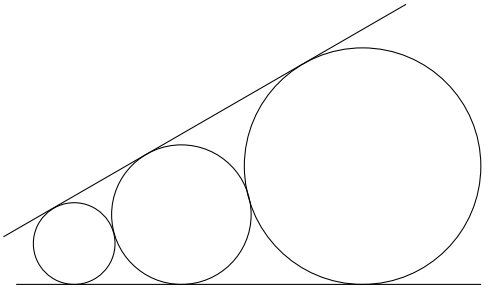
   and that

$$166 - 159 = 7 \text{ and } 94 - 81 = 13.$$

   Thus, 34 117 is a number that can be written as the sum of two squares of positive integers in two ways, where the first integers occurring in each sum differ by 7 and the second integers differ by 13.

   What is the smallest integer with this property?

8. In the figure the 3 circles are tangent to one another and to the two lines. The radius of the largest circle is 720 units, and that of the smallest is 320 units.



   What is the radius of the middle circle?

9. An *autobiographical number* is a natural number whose first (i.e. leftmost) digit is the number of zeros in it, the second digits is the number of ones, and so on.

   What is the remainder when the smallest autobiographical number is divided by 1000?

10. Mr. and Mrs. Jones went to a party with four other couples. Some of the people shook hands when they met, but no partners shook hands with each other, no two people shook hands more than once, and no one shook hands with himself or herself. Later Mr. Jones asked everyone else how many times they had shaken hands and received a different answer from each person.

    How many times did Mrs. Jones shake hands?