The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

**AMO TRAINING SESSIONS**

# 1999 Australian Intermediate Mathematics Olympiad Problems

1. How many digits are there in the smallest number which is composed entirely of fives (e.g. 55555) and which is divisible by 99?

2. In the addition sum $TAP + BAT + MAN$, each letter represents a different digit and no first digit is zero.

   What is the smallest sum that can be obtained?

3. Chord $AB$ subtends an angle of 153° at the centre $O$ of a circle. Point $P$ is the point of trisection of the major arc $AB$ closer to $B$.

   If $Q$ is a point on the minor arc $AB$, find in degrees $\angle AQP$.

4. How many different integers $x$ satisfy the equation

$$(x^2 - 5x + 5)^{x^2-11x+30} = 1?$$

5. Which three-digit number has the greatest number of different factors?

6. My father had to wash some nappies in a strong bleach solution and wanted to rinse them so that they contained as small a concentration of bleach as possible. He can wring them out so there is just half a litre of liquid left. He wrings them out, adds 12 litres of water, mixes thoroughly, wrings them out, adds a further 8 litres of water, and mixes thoroughly, reducing the concentration of bleach to $\frac{1}{k}$ of its original concentration.

   Find $k$.

7. An integer $N$ has digit representation $(abc)$ such that

$$N = b(10c + b)$$

   with $b$ and $10c + b$ prime.

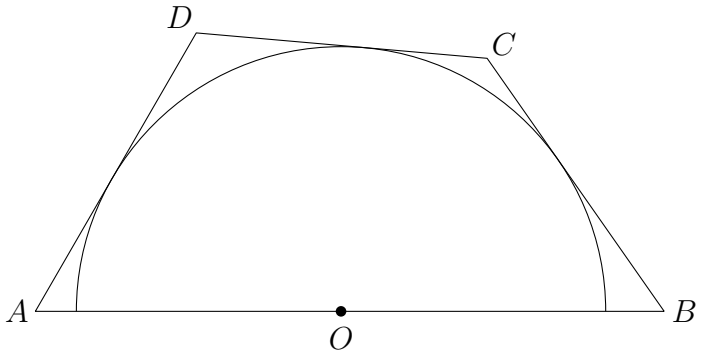
   Find $N$.

8. A *palindrome* is an integer which is the same number when its digits are reversed, e.g. 121 is a palindrome.

   Find an integer $n$ such that $n^2$ is a 6-digit palindrome.

9. A semicircle, centre $O$, is inscribed in quadrilateral $ABCD$ as shown, such that $AO = OB$.

   Prove that $AO^2 = AD \cdot BC$.




10. $N$ is the smallest positive integer such that the sum of the digits of $N$ is 18 and the sum of the digits of $2N$ is 27.

    Find $N$.

    *Investigation.*

    (a) $M$ is the largest positive integer containing no zeros such that the sum of the digits of $M$ is 18 and the sum of the digits of $2M$ is 27.

    Find $M$.

    (b) Find other numbers $N$ with the property that the sum of the digits of $N$ is 18 and the sum of the digits of $2N$ is 27.

    (c) Let $S(n)$ be the sum of the digits of $n$, where $n \in \mathbb{N}$.

    If $S(N) = a$ and $S(2N) = b$, find rules relating $a$ and $b$.

    If $N$ is a 4-digit number, what is the largest possible value of $a - b$ and for what values of $N$ does this occur?