The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2000 Australian Intermediate Mathematics Olympiad Problems

1. The integer n is the smallest positive multiple of 15, such that every digit of n is either 0 or 2.

   Compute $\frac{n}{15}$.

2. Evaluate the product

$$(\sqrt{2}+\sqrt{11}+\sqrt{13})(\sqrt{2}+\sqrt{11}-\sqrt{13})(\sqrt{2}-\sqrt{11}+\sqrt{13})(-\sqrt{2}+\sqrt{11}+\sqrt{13}).$$

3. Each of the interior angles of a heptagon (i.e. a 7-gon) is obtuse and the number of degrees in each angle is a multiple of 9, with no two angles equal.

   Find in degrees the sum of the largest two angles in the heptagon.

4. Briony takes a standard pack of 52 cards and throws some cards away. However, she makes sure she keeps all four aces among the remaining cards. She then selects four cards at random from these remaining cards.

   If the probability of her selecting the four aces is $\frac{1}{1001}$, how many cards did she throw away?

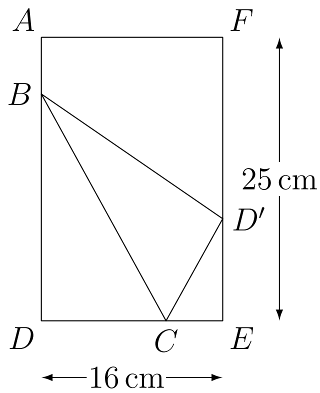
5. Let $(abcd)$ be a four-digit number with the following properties:

$$a+b+c+d=(ab)=c\times d,$$

   where (ab) denotes the digit representation of the number $10a+b$.

   What is the remainder when the largest number with these properties is divided by 1000?

6. A $16\,\mathrm{cm}\times 25\,\mathrm{cm}$ rectangular piece of paper is folded so that a corner touches the opposite side as shown, where $AB=5\,\mathrm{cm}$. Find BC^2.



7. Postumo is a stamp collector who specialises in stamps from Aruba, Mayotte and Svalbard. From time to time he rings his friends in these countries to talk about the latest stamp issues. The international phone codes for these countries are

   Aruba: 197, Mayotte: 169, and Svalbard: 47.

   So far, Postumo has collected a total of 30 stamps from these countries. He has more stamps from Aruba than from Mayotte, but fewer stamps from Aruba than from Svalbard. The number of his Svalbard stamps is more than four times but less than five times the number of his Aruban stamps. While this number is insufficient to determine the exact number of stamps from two countries in Postumo's collection, it is possible to determine the number of stamps from the remaining country.

   What is the result when you add this number to the country's international phone code?

8. A function $f : \mathbb{R} \to \mathbb{R}$ satisfies

$$f(2+x) = f(2-x) \text{ and } f(7+x) = f(7-x).$$

   If $x = 0$ is a solution of $f(x) = 0$, what is the smallest number of solutions that $f(x) = 0$ could have in the interval $-2000 \leq x \leq 2000$?

9. A trapezium is divided into four triangles by its diagonals.

   If X and Y are the areas of the triangles adjacent to the parallel sides, find the area of the trapezium in terms of X and Y.

10. For $n \in \mathbb{N}$, let $g(n)$ denote the number of ordered pairs (x, y) of positive integers such that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n}.$$

   (a) Find $g(10)$.

   (b) Find $g(2000)$.

   Investigation.

   (a) For $n \in \mathbb{N}$, let $h(n)$ denote the number of ordered pairs (x, y) of positive integers such that

$$\frac{2}{x} + \frac{1}{y} = \frac{1}{n}.$$

   Find $h(2000)$.

   (b) Find $g(p)$ if p is prime.