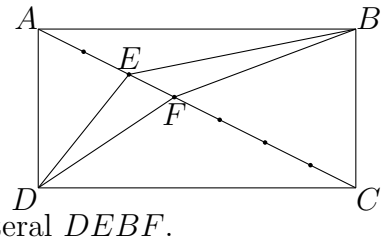
The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2007 Australian Intermediate Mathematics Olympiad Problems

1. Trevor's trailer has two wheels on its axle and carries a spare wheel. The three wheels are changed around from time to time. The three tyres have been worn for 25 000 km, 28 000 km and 31 000 km, respectively. How many thousand kilometres has Trevor's trailer travelled?

2. The rectangle shown has sides of length 28 and 15. The diagonal is divided into 7 equal parts.



   Find the area of the quadrilateral $DEBF$.

3. When 113 744 and 109 417 are divided by a 3-digit positive integer N, the remainders are 119 and 292, respectively. Find N.

4. ABC is a triangle with $AB = 85$, $BC = 75$ and $CA = 40$. A semicircle is tangent to AB and AC and its diameter lies on BC. Find the radius of the semicircle.

5. Find $x + y$ where x and y are non-zero solutions of the system of equations

$$y^2x = 15x^2 + 17xy + 15y^2$$
$$x^2y = 20x^2 + 3y^2.$$

6. When a positive integer N is written in base 4 it has three digits. When $3N$ is written in base 6 it also has three digits and has the same middle digit as N to base 4. Find the decimal sum of all such numbers N.

7. $x^2 - 19x + 94$ is a perfect square where x is an integer. Find the largest possible value of x.

8. A point P is marked inside a regular hexagon $ABCDEF$ so that $\angle BAP = \angle DCP = 50°$. Find $\angle ABP$.

9. Find a prime p with the property that for some larger prime q, both $2q - p$ and $2q + p$ are prime. Prove that there is only one such p.

10. In a triangle ADC, $DC = 65$ and altitudes DB and CE have lengths 33 and 63, respectively. Prove that the lengths of AB and AE cannot both be integers.

    Investigation

    Find AB and AE.

    In a triangle $A'D'C'$, $D'C' = 65k$ and altitudes $D'B'$ and $C'E'$ have lengths $33k$ and $63k$, respectively. Is there a value for k so that $A'B'$ and $A'E'$ are integers? If not, explain why. If so, find all such values of k.