The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2007 Australian Intermediate Mathematics Olympiad: Problems 1 and 2 with Solutions

1. Trevor's trailer has two wheels on its axle and carries a spare wheel. The three wheels are changed around from time to time. The three tyres have been worn for 25 000 km, 28 000 km and 31 000 km, respectively. How many thousand kilometres has Trevor's trailer travelled?

   Solution. Number the tyres 1, 2 and 3, so that the tyres have been worn 25 000 km, 28 000 km and 31 000 km, respectively, and let

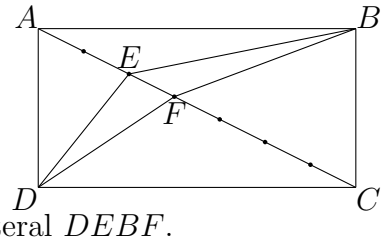
   $$\begin{aligned} x &= \text{thousands of km travelled on tyres 1 and 2} \\ y &= \text{thousands of km travelled on tyres 1 and 3} \\ z &= \text{thousands of km travelled on tyres 2 and 3.} \end{aligned}$$

   Then

   $$\begin{aligned} x + y &= 25 \\ x + z &= 28 \\ y + z &= 31 \\ \therefore 2(x + y + z) &= 25 + 28 + 31 \\ &= 84 \\ x + y + z &= 42 \end{aligned}$$

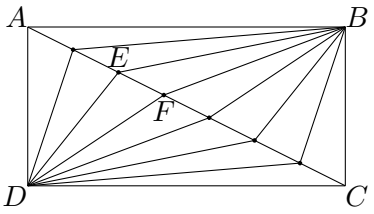
   The total distance travelled by Trevor's trailer is $x + y + z = 42$ thousand km.

2. The rectangle shown has sides of length 28 and 15. The diagonal is divided into 7 equal parts.



   Find the area of the quadrilateral $DEBF$.

   Solution.
   Join all the marks on the diagonal to B and D as shown.
   The seven triangles on one side of the diagonal have equal bases and the same height. So they have the same area.
   Hence the rectangle is divided into 7 quadrilaterals of equal area.
   Therefore, the area of $DEBF$ is



   $$\frac{28 \times 15}{7} = 4 \times 15 = 60.$$