The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

2008 Australian Intermediate Mathematics Olympiad Problems

1. Consider a circular sector of radius 360 which is one-sixth of a circle. A circle is drawn inside this sector so that its is tangent to the two radii and to the circular arc. Calculate the radius of this smaller circle.

2. Find the 3-digit number at the right-hand end of $1! + 2! + 3! + \cdots + 2008!$.

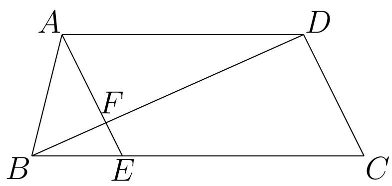
3. If $\dfrac{2}{35} = \dfrac{1}{x} + \dfrac{1}{y}$ and x, y are different positive integers, find the minimum value of $x + y$.

4. Find the largest prime factor of $7^{14} - 56 + 7^{13}$.

5. Each interior angle in a 16-sided convex polygon is an integer number of degrees. When arranged in ascending order of magnitude, these angles form an arithmetic progression. How many degrees are there in the largest interior angle in the polygon?

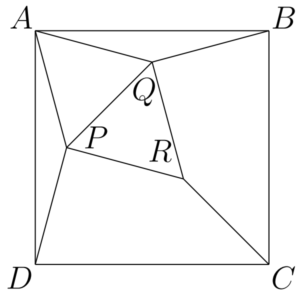
6. Impatient Imran always walks down the moving escalator outside his office. It moves at a constant but annoyingly slow speed. Once he got from top to bottom in 16 seconds taking 28 steps. Another time he got from top to bottom in 24 seconds taking 21 steps. How many steps high is the escalator?

7. In the trapezium $ABCD$, $AD \parallel BC$, $BD \perp DC$, the point F is chosen on diagonal BD so that $AF \perp BD$, and AF is extended to meet BC at the point E. If $AB = 41$, $BF = 9$ and the area of quadrilateral $FECD$ is 960, what is the length of AD?



8. Curious Kate calculated the sum of all positive integers each of which equals 101 times the sum of its digits. Find the remainder when her sum is divided by 1000.

9. $ABCD$ is a square. P, Q. R are points such that $\triangle APQ$ and $\triangle PQR$ are equilateral, and $AQ = QB$ and $AP = PD$. Prove that $RC = PD$.



10. Real numbers a, b, c, d, e are linked by the two equations:

$$e = 40 - a - b - c - d$$
$$e^2 = 400 - a^2 - b^2 - c^2 - d^2.$$

Determine the largest value for e.

Investigation

Find all integer solutions, if any, of the following pair of equations.

$$e = 30 - a - b - c - d$$
$$e^2 = 200 - a^2 - b^2 - c^2 - d^2.$$