The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

2011 Australian Intermediate Mathematics Olympiad Problems

1. Asha, Bree and Cala are three robots that are programmed to run athletic track races. When Asha runs a 400 m race she catches Bree at the finish line, if Bree starts 20 m ahead of Asha. Asha catches Cala at the finish line of a 1500 m race, if Cala has a 246 m start.

   Assuming each of the robots runs at constant speed, how many metres must Cala start ahead of Bree in an 800 m race, if they are to finish at the same time?

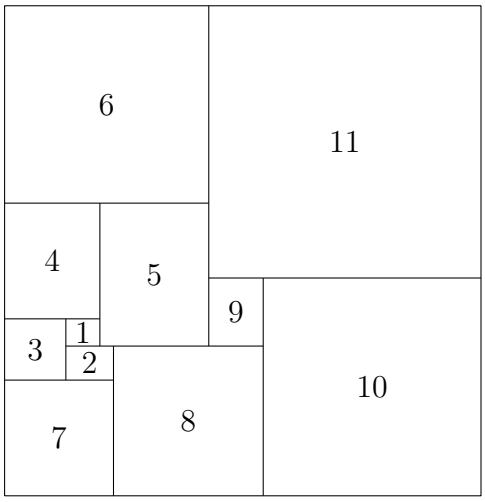
2. Find the number of solutions to the equation $3x + 7y = 2011$ for which both x and y are positive integers.

3. The square of the lengths of the diagonals of a rectangular prism are $\frac{4525}{36}, \frac{369}{4}, \frac{949}{9}$.

   Find the volume of the prism.

4. The large rectangle shown is divided into 11 squares, which are numbered as shown. Square 1 has perimeter 36.

   Given that the diagram is not to scale, find the perimeter of the large rectangle.



5. A number of points are placed on the circumference of a circle. Each pair of points, except for adjacent pairs are joined by a line segment. The total number of line segments is less than 100.

   What is the maximum number of line segments that could have been drawn?

6. There are a number of towns on a circular road. They are served by five buses. Each bus travels the entire circular road but stops at only five towns. For each pair of towns there is a bus that stops at both.

   Find the largest possible number of towns on the road.

7. Equilateral $\triangle ABC$ is inscribed in a circle. The point D is on minor arc AC of the circle. The line CD meets the line BA at E and the line AD meets the line BC at F. The length of CF is $21\sqrt{3}$ and the length of AE is $84\sqrt{3}$.

   Find the radius of the circle.

8. Polynomial $p(x)$ has non-negative integer coefficients.

   Given $p(5) = 2011$, find the minimum possible value of $p(4)$.

9. In a convex pentagon $ABCDE$ the sides AB, BC, CD, and DE have the same length and M is the mid-point of AE.

   If the interior angles at B and D are $90°$, prove $\angle BMD = 90°$.

10. Let a, b, c be three distinct positive integers. Show that not all of the three numbers $a + b$, $a + c$, $b + c$ are non-negative integer powers of 2, i.e. have the form 2^n for some non-negative integer n.

    Investigation

    Find all positive integers k for which not all three numbers $a + b$, $a + c$, $b + c$ are non-negative integer powers of k.