The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

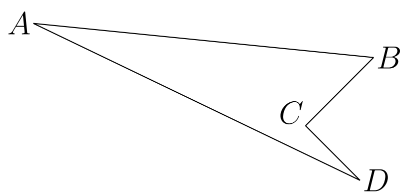
2012 Australian Intermediate Mathematics Olympiad Problems

1. Each letter in the grid represents a positive integer. The sum of any three consecutive entries of the grid is 164.

31	B	C	D	E	7	G	H	I

Find the value of H.

2. A yacht race takes place on the course depicted in the diagram. The race starts and finishes at A, with marker buoys at B, C, and D. Distance AB is 6 km, distance DA is 6.5 km, buoys B and C are 2 km apart, and buoy C is exactly the halfway mark for the race.



A yacht moving from buoy B to buoy C, has heading southwest, and moving from buoy C to buoy D, its heading is southeast.

If the area of water bounded by the course is $R\,\text{km}^2$, find the value of R.

3. Two identical bottles are filled with weak alcohol solutions. In one bottle, the ratio of the volume of alcohol to the volume of water is $1 : 25$. In the other bottle, the ratio of the volume of alcohol to the volume of water is $1 : 77$. If the contents of the bottles are mixed together, the ratio of the volume of alcohol to the volume of water is $1 : N$.

Find the value of N.

4. What is the maximum number of terms in a series of consecutive even positive integers whose sum is 1974?

5. Let $x, y \in \mathbb{N}$ such that $x^{5x} = y^y$.

What is the largest possible value for x?

6. The lengths of the sides of triangle T are 190, 323 and 399.

What is the length of the shortest altitude of T?

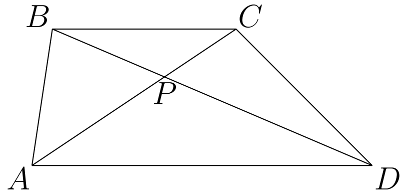
7. Non-zero real numbers x, y, z satisfy

$$\begin{aligned} xy &= 2(x+y) \\ yz &= 3(y+z) \\ zx &= 4(z+x). \end{aligned}$$

Determine $5x + 7y + 9z$.

8. $ABCD$ is a trapezium with $AD \parallel BC$. The area of $ABCD$ is 225. The area of $\triangle BPC$ is 49.

What is the area of $\triangle APD$?



9. The n^{th} triangular number is the sum of the first n positive integers. Let T_n denote the sum of the first n triangular numbers.

   Derive a formula for T_n and, hence or otherwise, prove that $T_n + 4T_{n-1} + T_{n-2} = n^3$.

10. A bag contains a certain number of 5c, 10c, 20c, 50c and \$1 coins of more than one denomination, e.g. if the bag contains nine 5c coins and one 20c coin, then it contains exactly ten coins and two denominations. Notice in the example that, if any coin is removed from the bag, then the remaining coins can be divided into three heaps of equal value.

    Determine all possible combinations of distinct denominations the bag may contain so that after removing any coin from the bag, its contents can be divided into three heaps of equal value.

    Investigation

    Suppose the bag contains a certain number of 5c, 10c, 20c, 50c, \$1 and \$2 coins of more than one denomination.

    Determine all possible combinations of distinct denominations the bag may contain so that after removing any coin from the bag, its contents can be divided into two heaps of equal value.