The University of Western Australia
DEPARTMENT OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

2013 Australian Intermediate Mathematics Olympiad Problems

1. Find the area in cm^2 of a rhombus whose side length 29 cm and whose diagonals differ in length by 2 cm.

2. How many 4-digit numbers are there whose digit product is 60?

3. A base 7 three-digit number has its digits reversed when written base 9.
   Find the decimal represntation of the number.

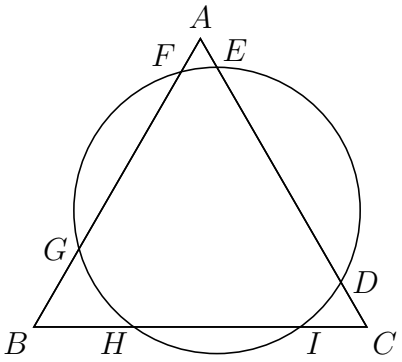
4. The prime numbers p, q, r satisfy the equations: $pq + pr = 80,\ pq + qr = 425$.
   Find the value of $p + q + r$.

5. How many pairs of 3-digit palindromes are there such that when they are added together, the result is a 4-digit palindrome.

6. ABC is an equilateral triangle with side length $2013\sqrt{3}$.
   Find the largest diameter for a circle in one of the regions between $\triangle ABC$ and its inscribed circle.

7. If $a, b, c, d \in \mathbb{N}$ have sum 63, what is maximum value of $ab + bc + cd$?

8. A circle meets the sides of an equilateral triangle ABC at six points D, E, F, G, H, I.
   If $AE = 4$, $ED = 26$, $DC = 2$, $FG = 14$, and the circle with diameter HI has area πb, find b.



9. A box contains some identical tennis balls. The ratio of the total volume of the tennis balls to the volume of empty space surrounding them in the box is $1 : k$, where $1 < k \in \mathbb{N}$.
   A prime number of tennis balls is removed from the box. Now the ratio of volume of tennis balls to volume of empty space in the box is $1 : k^2$.
   Find the number of tennis balls that were in the box originally.

10. I have a 1 m × 1 m square, which I want to cover with three discs of equal size, which are allowed to overlap.
    Show this is possible if the discs have diameter 1008 mm.

    Investigation

    Two discs of equal diameter cover a 1 m × 1 m square.
    Find their minimum diameter.