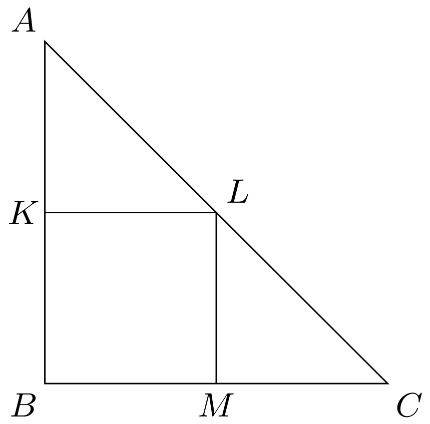
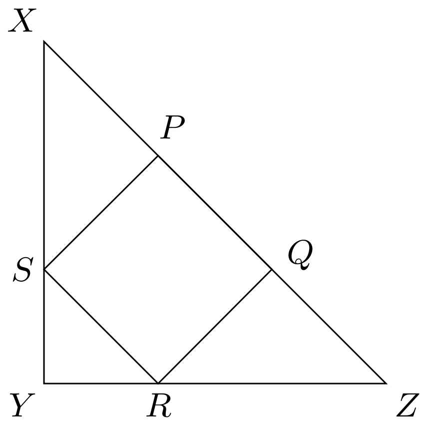
Time allowed: 4 hours. NO calculators are to be used.

Questions 1 to 8 only require their numerical answers all of which are non-negative integers less than 1000.
Questions 9 and 10 require written solutions which may include proofs.
The bonus marks for the Investigation in Question 10 may be used to determine prize winners.

1. In base b, the square of 24_b is 521_b. Find the value of b in base 10. [**2 marks**]

2. Triangles ABC and XYZ are congruent right-angled isosceles triangles. Squares $KLMB$ and $PQRS$ are as shown. If the area of $KLMB$ is 189, find the area of $PQRS$.





[**2 marks**]

3. Let x and y be positive integers that simultaneously satisfy the equations $xy = 2048$ and $\frac{x}{y} - \frac{y}{x} = 7.875$. Find x. [**3 marks**]

4. Joel has a number of blocks, all with integer weight in kilograms. All the blocks of one colour have the same weight and blocks of a different colour have different weights.

Joel finds that various collections of some of these blocks have the same total weight w kg. These collections include:

1. 5 red, 3 blue and 5 green;
2. 4 red, 5 blue and 4 green;
3. 7 red, 4 blue and some green.

If $30 < w < 50$, what is the total weight in kilograms of 6 red, 7 blue and 3 green blocks? [**3 marks**]

5. Let $\frac{1}{a} + \frac{1}{b} = \frac{1}{20}$, where a and b are positive integers. Find the largest value of $a + b$. [**4 marks**]

6. Justin's sock drawer contains only identical black socks and identical white socks, a total of less than 50 socks altogether.

If he withdraws two socks at random, the probability that he gets a pair of the same colour is 0.5. What is the largest number of black socks he can have in his drawer? [**4 marks**]

7. A *code* is a sequence of 0s and 1s that does not have three consecutive 0s. Determine the number of codes that have exactly 11 digits. [**4 marks**]

8. Determine the largest integer n which has at most three digits and equals the remainder when n^2 is divided by 1000. [**4 marks**]

9. Let $ABCD$ be a trapezium with $AB \parallel CD$ such that

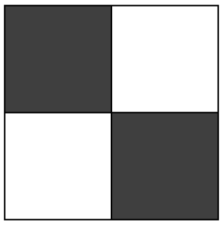
(i) its vertices A, B, C, D, lie on a circle with centre O,

(ii) its diagonals AC and BD intersect at point M and $\angle AMD = 60°$,

(iii) $MO = 10$.

Find the difference between the lengths of AB and CD. [**5 marks**]

10. An $n \times n$ grid with $n > 1$ is covered by several copies of a 2×2 square tile as in the figure below. Each tile covers precisely four cells of the grid and each cell of the grid is covered by at least one cell of one tile. The tiles may be rotated 90 degrees.



(a) Show there exists a covering of the grid such that there are exactly n black cells visible.
(b) Prove there is no covering where there are less than n black cells visible.
(c) Determine the maximum number of visible black cells. [**4 marks**]

Investigation

(i) Show that, for each possible pattern of 3 black cells and 6 white cells on a 3×3 grid, there is a covering whose visible cells have that pattern. [**1 bonus mark**]

(ii) Explain why not all patterns of 4 black cells and 12 white cells on a 4×4 grid can be achieved with a covering in which each new tile must be placed on top of all previous tiles that it overlaps. [**1 bonus mark**]

(iii) Determine the maximum number of visible black cells for a covering of an $n \times m$ grid where $1 < n < m$. [**2 bonus marks**]