CHALLENGE STATISTICS – INTERMEDIATE

Mean Score/School Year/Problem

Year	Number of Students	Mean						
		Overall	Problem					
			1	2	3	4	5	6
9	1787	12.9	2.8	2.6	2.2	2.2	2.6	1.6
10	1061	14.5	2.9	2.8	2.4	2.4	2.9	2.0
*ALL YEARS	2877	13.6	2.9	2.7	2.3	2.3	2.7	1.8

Please note:* This total includes students who did not provide their school year.

Score Distribution %/Problem

Score	Challenge Problem					
	1 Indim Integers	2 Digital Sums	3 Coin Flips	4 Jogging	5 Folding Fractions	6 Crumbling Cubes
Did not attempt	2%	5%	6%	5%	10%	16%
0	4%	9%	13%	5%	9%	23%
1	11%	9%	13%	22%	11%	13%
2	18%	20%	22%	28%	11%	18%
3	28%	20%	25%	23%	25%	19%
4	37%	37%	21%	17%	34%	10%
Mean	2.9	2.7	2.3	2.3	2.7	1.8
Discrimination Factor	0.6	0.7	0.7	0.6	0.8	0.7

Please note:

The discrimination factor for a particular problem is calculated as follows:

(1) The students are ranked in regard to their overall scores.

(2) The mean score for the top 25% of these overall ranked students is calculated for that particular problem including no attempts. Call this mean score the 'mean top score'.

(3) The mean score for the bottom 25% of these overall ranked students is calculated for that particular problem including no attempts. Call this mean score the 'mean bottom score'.

(4) The discrimination factor = $\dfrac{\text{mean top score} - \text{mean bottom score}}{4}$

Thus the discrimination factor ranges from 1 to –1. A problem with a discrimination factor of 0.4 or higher is considered to be a good discriminator.

AUSTRALIAN INTERMEDIATE MATHEMATICS OLYMPIAD

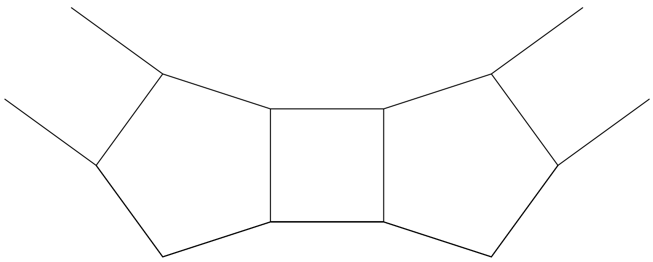
Time allowed: 4 hours. NO calculators are to be used.

Questions 1 to 8 only require their numerical answers all of which are non-negative integers less than 1000.
Questions 9 and 10 require written solutions which may include proofs.
The bonus marks for the Investigation in Question 10 may be used to determine prize winners.

1. Find the smallest positive integer x such that $12x = 25y^2$, where y is a positive integer. [**2 marks**]

2. A 3-digit number in base 7 is also a 3-digit number when written in base 6, but each digit has increased by 1. What is the largest value which this number can have when written in base 10? [**2 marks**]

3. A ring of alternating regular pentagons and squares is constructed by continuing this pattern.



How many pentagons will there be in the completed ring? [**3 marks**]

4. A sequence is formed by the following rules: $s_1 = 1, s_2 = 2$ and $s_{n+2} = s_n^2 + s_{n+1}^2$ for all $n \geq 1$.

What is the last digit of the term s_{200}? [**3 marks**]

5. Sebastien starts with an 11×38 grid of white squares and colours some of them black. In each white square, Sebastien writes down the number of black squares that share an edge with it.

Determine the maximum sum of the numbers that Sebastien could write down. [**3 marks**]

6. A circle has centre O. A line PQ is tangent to the circle at A with A between P and Q. The line PO is extended to meet the circle at B so that O is between P and B. $\angle APB = x^\circ$ where x is a positive integer. $\angle BAQ = kx^\circ$ where k is a positive integer. What is the maximum value of k? [**4 marks**]

7. Let n be the largest positive integer such that $n^2 + 2016n$ is a perfect square. Determine the remainder when n is divided by 1000. [**4 marks**]

8. Ann and Bob have a large number of sweets which they agree to share according to the following rules. Ann will take one sweet, then Bob will take two sweets and then, taking turns, each person takes one more sweet than what the other person just took. When the number of sweets remaining is less than the number that would be taken on that turn, the last person takes all that are left. To their amazement, when they finish, they each have the same number of sweets.

They decide to do the sharing again, but this time, they first divide the sweets into two equal piles and then they repeat the process above with each pile, Ann going first both times. They still finish with the same number of sweets each.

What is the maximum number of sweets less than 1000 they could have started with? [**4 marks**]