AUSTRALIAN INTERMEDIATE MATHEMATICS OLYMPIAD

1. The number x is 111 when written in base b, but it is 212 when written in base $b-2$. What is x in base 10?

[**2 marks**]

2. A triangle ABC is divided into four regions by three lines parallel to BC. The lines divide AB into four equal segments. If the second largest region has area 225, what is the area of ABC?

[**2 marks**]

3. Twelve students in a class are each given a square card. The side length of each card is a whole number of centimetres from 1 to 12 and no two cards are the same size. Each student cuts his/her card into unit squares (of side length 1 cm). The teacher challenges them to join all their unit squares edge to edge to form a single larger square without gaps. They find that this is impossible.

Alice, one of the students, originally had a card of side length a cm. She says, 'If I don't use any of my squares, but everyone else uses their squares, then it is possible!'

Bob, another student, originally had a card of side length b cm. He says, 'Me too! If I don't use any of my squares, but everyone else uses theirs, then it is possible!'

Assuming Alice and Bob are correct, what is ab?

[**3 marks**]

4. Aimosia is a country which has three kinds of coins, each worth a different whole number of dollars. Jack, Jill, and Jimmy each have at least one of each type of coin. Jack has 4 coins totalling \$28, Jill has 5 coins worth \$21, and Jimmy has exactly 3 coins. What is the total value of Jimmy's coins?

[**3 marks**]

5. Triangle ABC has $AB = 90$, $BC = 50$, and $CA = 70$. A circle is drawn with centre P on AB such that CA and CB are tangents to the circle. Find $2AP$.

[**3 marks**]

6. In quadrilateral $PQRS$, $PS = 5$, $SR = 6$, $RQ = 4$, and $\angle P = \angle Q = 60°$. Given that $2PQ = a + \sqrt{b}$, where a and b are unique positive integers, find the value of $a + b$.

[**4 marks**]

7. Dan has a jar containing a number of red and green sweets. If he selects a sweet at random, notes its colour, puts it back and then selects a second sweet, the probability that both are red is 105% of the probability that both are red if he eats the first sweet before selecting the second. What is the largest number of sweets that could be in the jar?

[**4 marks**]

8. Three circles, each of diameter 1, are drawn each tangential to the others. A square enclosing the three circles is drawn so that two adjacent sides of the square are tangents to one of the circles and the square is as small as possible. The side length of this square is $a + \dfrac{\sqrt{b} + \sqrt{c}}{12}$ where a, b, c are integers that are unique (except for swapping b and c). Find $a + b + c$.

[**4 marks**]

9. Ten points $P_1, P_2, \ldots, P_{10}$ are equally spaced around a circle. They are connected in separate pairs by 5 line segments. How many ways can such line segments be drawn so that only one pair of line segments intersect?

[**5 marks**]

10. *Ten-dig* is a game for two players. They try to make a 10-digit number with all its digits different. The first player, A, writes any non-zero digit. On the right of this digit, the second player, B, then writes a digit so that the 2-digit number formed is divisible by 2. They take turns to add a digit, always on the right, but when the nth digit is added, the number formed must be divisible by n. The game finishes when a 10-digit number is successfully made (in which case it is a *draw*) or the next player cannot legally place a digit (in which case the other player *wins*).

Show that there is only one way to reach a draw.

[**5 marks**]

Investigation

Show that if A starts with any non-zero even digit, then A can always win no matter how B responds.

[**4 bonus marks**]