AUSTRALIAN MATHS TRUST

2019 Australian Intermediate Mathematics Olympiad

Questions

1. Gaston and Jordon are two budding chefs who unfortunately always misread the cooking time required for a recipe. For example, if the required cooking time is written as 1:32, meaning 1 hour and 32 minutes, Jordon reads it as 132 minutes while Gaston reads it as 1.32 hours. For one particular recipe, the difference between Jordon's and Gaston's misread times is exactly 90 minutes. What is the actual cooking time in minutes?

[**2 marks**]

2. An integer has 3 digits in base 10. When it is interpreted in base 6 and multiplied by 4, the result is 2 more than 3 times the same number when it is interpreted in base 7. What is the largest integer, in base 10, that has this property?

[**2 marks**]

3. Let $ABCD$ be a square with side length 24. Let P be the point on side AB with $AP = 8$, and let AC and DP intersect at Q. Determine the area of triangle CQD.

[**3 marks**]

4. Jenny has a large bucket of 1000 marbles, of equal size. At least one is red and the rest are green. She calculates that if she picks out two marbles at random, the probability that both are red is the same as the probability that they are different colours. How many red marbles are in the bucket?

[**3 marks**]

5. A beetle starts at corner A of a square $ABCD$. Each minute, the beetle moves along exactly one side of the square from one corner of the square to another corner. For example, in the first two minutes, the beetle could have one of 4 walks: ABA, ADA, ABC, ADC. How many walks can the beetle have from corner A to corner C in 10 minutes?

[**3 marks**]

6. A leaky boat already has some water on board and water is coming in at a constant rate. Several people are available to operate the manual pumps and, when they do, they all pump at the same rate. Starting at a given time, five people would take 10 hours to pump the boat dry, while 12 people would take only 3 hours. How many people are required to pump it dry in 2 hours from the given time?

[**4 marks**]

PLEASE TURN OVER THE PAGE FOR QUESTIONS 7, 8, 9, AND 10

7. A triangle PQR is to be constructed so that the perpendicular bisector of PQ cuts the side QR at N and the line PN splits the angle QPR into two angles, not necessarily of integer degrees, in the ratio $1:22$. If $\angle QPR = p$ degrees, where p is an integer, find the maximum value of p.

[**4 marks**]

8. Find the largest number which will divide $a^5 + b^5 - ab(a^3 + b^3)$ for *all* primes a and b greater than 5.

[**4 marks**]

9. In a large botanic garden, there is a long line of 999 regularly spaced positions where plants may grow. At any time, there is at most one plant in each position. In the first year, only the middle position has a plant. In the nth year, for all $n \geq 2$, there is a plant at position X if and only if, in the previous year, the total number of plants in position X and either side of X is precisely one. Determine, with proof, the total number of plants in the 128th year.

[**5 marks**]

10. An even number of teams play in a round robin tournament defined as follows. Each team plays every other team exactly once. The matches are arranged in rounds. Each team plays exactly once in each round. We assume there are no draws.

(a) Prove that at the end of round 2, the number of teams that have won two matches equals the number of teams that have lost two matches.

(b) Find the least number of teams such that at the end of round 3, the number of teams that have won three matches may not equal the number of teams that have lost three matches.

(c) Prove that, at the end of round 4, the number of teams that have won 4 matches equals the number of teams that have lost 4 matches, if and only if the number of teams that have won exactly 3 matches equals the number of teams that have lost exactly 3 matches.

[**5 marks**]

Investigation

Prove that for all even $k \geq 4$, there is a round robin tournament of k teams in which the number of teams that have won $k-1$ matches does not equal the number of teams that have lost $k-1$ matches.

[**3 bonus marks**]