QUESTIONS

1. A teacher asks Annie, Basil, Cara, Dave, and Elli, in that order, to each multiply a pair of (possibly equal) single digits. Except for Annie, each of them gets a product that is 50% more than the previous student. What product did *Dave* get?

[**2 marks**]

2. How many 4-digit palindromes are divisible by 99? (A *palindrome* is a number, not starting with 0, that is the same when its digits are written in reverse order.)

[**2 marks**]

3. Label the vertices of a square A, B, C, D anticlockwise. A line ℓ passes through B and intersects the side AD. If A is 5 cm from ℓ and C is 7 cm from ℓ, find the area of $ABCD$ in square centimetres.

[**3 marks**]

4. Three students Andrew, Louise and Elaine attempted 100 mathematics problems. Each of them solved exactly 60 problems. We call a problem *easy* if all three students solved it and *difficult* if only one student solved it. Given each problem was solved by at least one student, find the difference between the number of difficult problems and the number of easy problems.

[**3 marks**]

5. Fiona wants 6 red, 3 green and 3 yellow buttons on her dress. The buttons are indistinguishable except for colour. All buttons must be placed along a straight vertical line with no two neighbouring buttons of the same colour. How many ways could Fiona order the buttons?

[**3 marks**]

6. Three positive integers a, b, c with $a > c$ satisfy the following equations:

$$\begin{aligned} ac + b + c &= bc + a + 66 \\ a + b + c &= 32 \end{aligned}$$

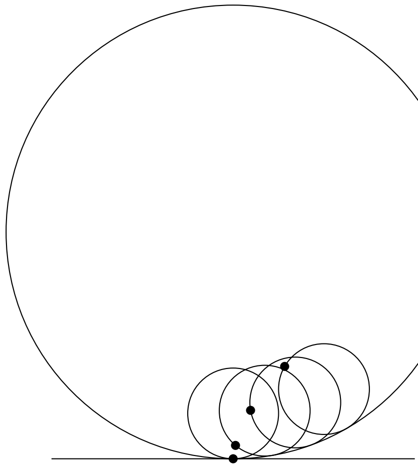
What is the value of abc?

[**4 marks**]

7. A positive integer is *positively assorted* if each of its digits are different and none are zero. Find the largest prime that divides the sum of all 4-digit positively assorted integers.

[**4 marks**]

8. A ride at an amusement park consists of a small circle which rotates inside a large circle. The large circle, of radius 100 metres, is tangent to the ground at its lowest point, and remains fixed in place throughout the ride. The smaller circle has radius 20 metres. It is initially tangent to the larger circle at its lowest point. Ada sits in the ride at this tangent point, on a seat attached to the smaller circle. When the ride starts, the smaller circle rolls around the larger circle without slipping, so that the circles always remain tangent, and Ada rotates with the smaller circle, as illustrated below.



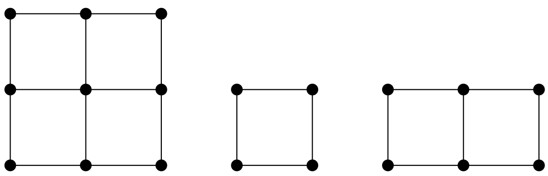
When the centre of the smaller circle has rotated $120°$ around the centre of the larger circle, how far in metres is Ada off the ground?

[**4 marks**]

9. Find all non-decreasing sequences of real numbers $a_1, a_2, a_3, \ldots$, such that $a_{2n} = 3a_n$ and $a_{3n} = 5a_n$ for all positive integers n.

[**5 marks**]

10. Let $n \geq m \geq 2$ and consider an $m \times n$ grid of unit squares. Such grids will contain many rectangles whose vertices are points on the grid, and whose edges are lines from the grid. For example, a 2×2 rectangular grid contains exactly three non-congruent rectangles as shown below.



Find all $m \times n$ grids that contain exactly 100 non-congruent rectangles.

[**5 marks**]

Investigation

Let $n \geq m \geq 2$. Describe the set of positive integers k for which there is an $m \times n$ grid containing k non-congruent rectangles.

[**3 bonus marks**]