AUSTRALIAN MATHS TRUST | AIMO AUSTRALIAN INTERMEDIATE MATHEMATICS OLYMPIAD

Thursday 12 September 2024

QUESTIONS

1. Two positive integers, a and b, satisfy the equation

$$\frac{2^{24}+2^{21}+2^{21}+2^{21}}{2024}=\frac{2^a}{23^b}.$$

Find the value of $a+b$.

[**2 marks**]

2. The area of a right-angled triangle is 120 and each of its three sides has integer length.

Find the length of its hypotenuse.

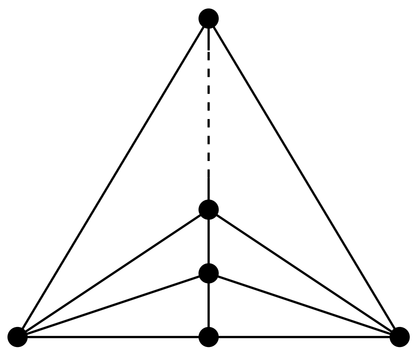
[**2 marks**]

3. Quadrilateral $ABCD$ has perimeter 60 cm and AB has length 9 cm. Sides AD and BC are parallel with distance 7 cm between them. Points E and F are chosen on sides BC and AD respectively so that the line EF divides $ABCD$ into two quadrilaterals with equal perimeters and equal areas.

What is the area of quadrilateral $ABCD$?

[**3 marks**]

4. In the following figure, all the central dots are collinear and the bottom three dots are collinear. There are exactly 2024 triangles that have dots as vertices (no three collinear dots form a triangle).



How many dots are there?

[**3 marks**]

5. Three positive integers a, b, c satisfy

$$a^2+b-c=100 \quad (1)$$
$$a+b^2-c=124 \quad (2)$$

Find $a+b+c$.

[**3 marks**]

PLEASE TURN OVER THE PAGE FOR QUESTIONS 6, 7, 8, 9, 10

6. Let $ABCD$ be a parallelogram in which the bisector of angle ABC intersects AD at P. Given $PD = 65$ and $BP = 78 = PC$, find AB.

[**4 marks**]

7. A survey of N people was taken to determine which of the different types A, B, C of screen entertainment they used, if any. The survey found:

- 50 people used B
- 61 did not use A
- 13 did not use C
- 74 used at least two of A, B, C.

Find the minimum value of N.

[**4 marks**]

8. A *binary sequence* is a sequence in which each term is 0 or 1.
A binary sequence $(a_0, a_1, \ldots, a_{n-1}, a_n)$ is *dyadic* if it has an even number of terms and $a_0 + a_n = a_1 + a_{n-1} = a_2 + a_{n-2} = \cdots = 1$.
For example, (1, 0, 0, 1, 1, 0) is a dyadic sequence of length 6.

How many binary sequences are there of length 16 that are *not* dyadic, but are composed of 4 consecutive dyadic sequences each of length 4?

[**4 marks**]

9. Denote the number of positive factors of an integer n by $F(n)$.
For example, the positive factors of 12 are 1, 2, 3, 4, 6, 12, so $F(12) = 6$.
In addition, the positive factors of 6 are 1, 2, 3, 6, so $F(F(12)) = 4$.

If $F(F(n)) = 3$ and $F(F(6n)) = 7$, find $F(n)$.

[**5 marks**]

10. Suppose that a number of 5×7 rectangles are placed on a large grid of unit squares so that each rectangle covers exactly 35 of the unit squares. The rectangles can be placed either horizontally or vertically and they may overlap.

Find the largest integer N for which it is *not* possible to cover exactly N unit squares in this way.

[**5 marks**]

Investigation

Suppose instead we have rectangles of a single size $3 \times n$, for some positive integer $n \geq 6$. Placing them on the grid as above, what is the largest integer N for which it is *not* possible to cover exactly N unit squares?

[**4 bonus marks**]

Optiver National sponsor of the Australian Informatics and Mathematical Olympiad programs

The Olympiad program is supported by the Australian Government Department of Industry, Science and Resources through the Science Competitions: Mathematics and Informatics Olympiads grant opportunity.

© 2024 Australian Mathematics Trust