

## Index

- $\forall$  for all, 11
- $\lceil \cdot \rceil$  ceiling, 142
- ' complement, 10
- $\sim$  congruent, 105
- $\nmid$  contradiction, 15
- | divides, 47
- $:$  divisible by, 47
- $\parallel$  exactly divides, 48
- $\emptyset$  empty set, 10
- $\exists$  there exists, 11
- $\lfloor \cdot \rfloor$  floor, 142
- 0hash
  - # ...
  - number of ..., 42
- $\iff$  if and only if, 13
- $\rightarrow$  implies (logic), 10
- $\implies$  implies (mathematics), 13
- $\in$  is an element of, 10
- $\cap$  intersection, 10
- $\| \cdot \|$  length of vector, 133
- $\leftrightarrow$  logically equivalent, 10
- $\neg$  negation, 11
- $\nmid$  not divide, 47
- $\parallel$  parallel, 132
- $\varphi$  Euler's totient function, 61
- $\sim$  similar, 108
- $\subset$  is a proper subset of, 10
- $\subseteq$  is subset of, 10
- $\setminus$  take (set subtraction), 10
- $\cup$  union, 10
- $\vee$  vee logic operator, 11
- $\wedge$  wedge logic operator, 11
- AA Rule
  - triangle similarity, 108
- abelian group, 137
  - axioms, 137
  - definition, 137
- absolute value, 140
- acute, 117
- Addendo, 120
- addition principle
  - counting, 41
- adjacent, 161
- algebra of limits, 147
- alternate angles, 108
- Alternate Segment Theorem
  - Tangent-chord Theorem, 115
- alternating sum of digits of  $N$ 
  - $A(N)$ , 59
- altitude, 117
  - relative to base, 112
- AM
  - 1-Power Mean, 96
  - Arithmetic Mean, 91
- AM-GM, 90
- AM-GM-HM, 91
- AM-HM, 92
- antiparallel, 132

- antisymmetry, 20
- apex
  - relative to base, 112
- arc, 117, 162
- area notation, 112
- Arithmetic Mean
  - AM, 91
- arithmetic series, 81
- ASA Rule
  - triangle congruence, 106
- associativity, 137
  - of vector  $+$ , 134
- asymmetry, 20
- $B_\delta(x)$ 
  - ball of radius  $\delta$  and centre  $x$ , 147
- $B_\delta^*(x)$ 
  - punctured ball of radius  $\delta$ , centre  $x$ , 147
- Bézout's Lemma, 54
- ball, 147
  - punctured, 147
- base, 112, 145
- binary relations, 19
- binomial coefficient
  - $\binom{n}{r}$ , 40
  - properties, 41
- binomial expansion, 81
- Binomial Theorem, 40, 80
- bipartite graph, 163
- Bolzano, 150
- bounded above, 35
- bounded below, 35
- Bowtie Theorem, 122
- bridge, 163
- Cauchy sequence, 139
- Cauchy-Schwarz Inequality, 92
- ceiling, 142
- centre of mass, 117
- centroid, 117
- Ceva's Lemma, 120
- Ceva's Theorem, 120
- cevian, 117, 120
- change of base rule, 146
- Chebyshev Inequality, 96
- Chinese Remainder Theorem, 60
- chord, 117, 119
- chromatic number, 163
- circuit, 162
- circumcentre, 116, 117
- circumcircle, 116, 117
- circumradius, 116
- circumscribed, 116
- clique, 163
- clopen, 140
- closed, 140
- closed walk, 162
- closure, 137
- codomain
  - of function, 141
- coefficient
  - of polynomial, 31
- collection, 7
- collinear, 117
- colouring, 163
  - $n$ -colouring, 163
- common difference, 79
- common ratio, 79
- commutativity, 137
  - of dot product, 136
  - of vector  $+$ , 134
- Comparison Test, 153
- compatibility
  - w.r.t. addition, 138
  - w.r.t. multiplication, 138
- complement, 10
  - of a graph, 162
- complementary angles, 117
- complete, 139
- complete graph, 163
- complete induction
  - polypus version, 27
- completing the square, 35
- component, 163
- composite, 48
- composition, 142
- concurrent, 117
- congruence modulo  $m$ , 58
- congruent, 58, 105, 117
- connected, 163
- constant coefficient, 31
- constant polynomial, 33
- constant term, 31

- continuous, 149
- contrapositive, 13
- converges, 151, 152
- converse, 13
- convex, 117
- coprime, 55
- corollary, 13
- corresponding angles, 108
- Cosine Rule, 110
- counting
  - addition principle, 41
  - multiplication principle, 41
- cut-vertex, 163
- cycle, 162
- cyclic, 117
  
- $\partial$ 
  - degree of polynomial, 32
- decagon
  - 10-gon, 119
- decimal representation, 59
- decreasing, 151
  - function, 21
- defined, 142
- degenerate, 77, 82
- degree, 32, 161
  - of polynomial, 31
- degree of polynomial
  - $\partial$ , 32
- degree sequence, 164
- diameter, 117, 163
- digraph, 162
- Diophantine equation
  - linear, 55
- direct proof, 13
- directed edge, 162
- directed segment convention, 122, 128
- discriminant, 34
- distance, 140, 163
- distribution, 134
- distribution of  $\cdot$  over  $+$ 
  - for vectors, 136
- distributive law
  - field, 137
- div
  - computing, 59
  - diverge, 152
  - divergent, 151
  - divides, 44, 47
    - exactly, 48
    - transitivity property, 48
  - divisible by, 47
  - division, 139
  - division algorithm, 47
    - for integers, 47
    - for polynomials, 33
  - division table
    - Euclidean Algorithm, 54
  - divisor, 47
  - divisors, 47
  - dodecagon
    - 12-gon, 119
  - $\text{dom}(f)$ 
    - domain of function  $f$ , 142
  - domain, 21
    - of function, 141
  - dot product, 136
  - dummy variable, 78, 83
  
  - edge, 117, 161
  - element, 10
  - empty set, 10
  - endpoints, 161
  - ends, 161
  - equal
    - for polynomials, 32
  - equation, 31
  - equilateral
    - triangle, 117
  - equivalence relation, 19
  - equivalent
    - vectors, 131
  - essential singularity, 150
  - Euclid's Lemma, 49, 67
  - Euclidean Algorithm, 53
    - division table, 54
  - Euler circuit, 162, 165
  - Euler line, 117, 121
  - Euler multigraph, 162
  - Euler tour, 162
  - Euler trail, 162
  - Euler's Theorem, 62

- in geometry, 124
- Euler's totient function
  - $\varphi$ , 61
- even vertex, 162
- exactly divides, 48
- excentres, 127
- excircles, 127
- existential quantifier
  - $\exists$ , 11
- explicit relation, 79
- exponentiation convention, 66
- exradii, 127
- Extended Euclidean Algorithm, 56
- Extended Pigeon-Hole Principle, 73
- external angle bisector, 127
- Extreme Value Theorem, 150
- factor
  - of integer, 48
  - of polynomial, 33
- factor over, 34
- Factor Theorem, 33
- factorial, 83
- Fermat's Little Theorem, 63, 67
- field, 137
  - definition, 137
- floor, 142
- forest, 163
- full order, 20
- function, 141
  - codomain, 141
  - domain, 141
  - image, 141
  - range, 141
- functional equation, 154
- Fundamental theorem of arithmetic, 49
- gcd
  - greatest common divisor, 53
- Geometric Mean
  - GM, 91
- geometric series, 152
- Gergonne point, 127
- GM
  - 0-Power Mean, 96
  - Geometric Mean, 91
- graph, 142, 161
  - $(p, q)$  graph, 161
- greatest common divisor, 53
- group
  - inverse, 137
- Hamiltonian circuit, 162
- Hamiltonian multigraph, 162
- Hamiltonian path, 162
- Handshaking Lemma, 164
- Harmonic Mean
  - HM, 91
- harmonic series, 152
- hcf
  - highest common factor, 53
- heptagon
  - 7-gon, 119
- Heron's Theorem, 113
- hexagon
  - 6-gon, 119
- highest common factor, 53
- HM
  - 1-Power Mean, 96
  - Harmonic Mean, 91
- homothetic, 125
- Horner's Method, 37
- hypotenuse, 117
- identity, 16, 19, 137, 144
  - under vector  $+$ , 134
- if and only if, 13
- iff, 13
- image, 141
  - of function, 141
- implies, 10, 13
- incentre, 116, 118
- incident, 161
- incircle, 116, 118
- increasing, 151
  - function, 21
- independent events, 41
- index, 77
- index laws, 145
- indices, 77
- injective, 143
- inradius, 116, 118
- inscribed, 116
- Integers

- $\mathbb{Z}$ , 8
- Intermediate Value Theorem, 150
- intersection, 10
- interval, 140
- into
  - (codomain of function), 141
- invariant, 159
- inverse
  - of group element, 137
- inverse function, 144
- inverses
  - under vector  $+$ , 134
- involution, 158
- irreflexivity, 20, 138
- isomorphic, 162
- isosceles, 106, 118
- jump discontinuity, 150
- Königsberg bridge problem, 165
- lcm
  - lowest common multiple, 53
- leading coefficient
  - of polynomial, 31
- leading term
  - of polynomial, 31
- leaf, 162
- least common multiple, 53
- lemma, 13
- length, 163
  - of vector, 133
- limit, 139, 147, 151
  - at infinity, 148
  - infinity, at infinity, 149
  - of sequence, 151
- Limit Comparison Test, 153
- line, 105, 118
- line segment, 105, 118
- linear Diophantine equation, 55
- locus, 118
- log laws, 146
- logarithm, 145
- logically equivalent, 10
- loop, 161
- lowest common multiple, 53
- maps, 142
  - action of function, 141
  - maps to
    - action of function, 142
  - mathematical induction, 17, 23
    - complete induction, 27
    - secondary induction, 27
    - simple induction, 27
    - strong induction, 27
  - maximum value, 35
  - medial triangle, 118, 121
  - median, 118
  - minimum value, 35
  - mod
    - computing, 59
  - modulo, 58
  - monic
    - polynomial, 31
  - monotone, 151
  - monotonic, 158
  - multigraph, 161
  - multiple, 47
  - multiplication principle
    - counting, 41
  - multiplicative, 62
  - mutually exclusive events, 41
- $\mathbb{N}$ 
  - set of Natural Numbers, 8
- $n$ -gon, 119
  - 10-gon (decagon), 119
  - 12-gon (dodecagon), 119
  - 3-gon (triangle), 119
  - 4-gon (quadrilateral), 119
  - 5-gon (pentagon), 119
  - 6-gon (hexagon), 119
  - 7-gon (heptagon), 119
  - 8-gon (octagon), 119
  - 9-gon (nonagon), 119
- Natural Numbers
  - $\mathbb{N}$ , 8
  - necessary and sufficient condition, 13
  - necessary condition, 13
  - negation, 10, 11
  - negative, 138
  - neighbourhood, 147
    - punctured, 147

- nhd, 147
- nine-point circle, 118, 122
- non-edge, 161
- nonagon
  - 9-gon, 119
- nonnegative, 138
- nonpositive, 138
- not divide, 47
- number line, 137
- number of, 42
- octagon
  - 8-gon, 119
- odd vertex, 162
- one, 137
- one-to-one, 143
- onto, 142
- open, 140
- opposite, 105
- order, 161
  - of a group, 62
  - of an element, 62
- ordered field, 138
- orthic triangle, 118, 121, 127
- orthocentre, 117, 118
- orthogonal, 118
- over, 31
- $\mathcal{P}(P, K)$ 
  - power of point  $P$  w.r.t. circle  $K$ , 122
- $p$ -series, 152
- PAP Rule
  - triangle similarity, 108
- PAP, PPP, AA, PPA Rules, 108
- parallel, 108, 118, 132
- parallelogram, 118
- Parallelogram Law
  - for vectors, 132
- parity, 36
- partial order, 20
- partial sum, 152
- Pascal's triangle, 41
- path, 162
  - $u$ - $v$  path, 162
- pedal point, 118
- pedal triangle, 118
- pentagon
  - 5-gon, 119
- perfect square, 36
- periodic, 158
- permutation, 41
- perpendicular, 118
- Pigeon-Hole Principle, 73
- planar, 163
- plane figure, 118
- PMI
  - Principle of Mathematical Induction, 24
- $PM_k$ 
  - Power Mean, 96
- point, 105
- point of contact, 119
- pointwise, 142
- polygon
  - $n$ -gon, 119
- polynomial, 31
- polynomial equation, 31
- position vector, 131
- positive, 138
- power, 122
- Power Mean
  - $PM_k$ , 96
  - Power Mean (Hölder Mean), 96
- power of a point
  - $\mathcal{P}(P, K)$ , 122
- PPA Rule
  - triangle similarity, 108
- PPP Rule
  - triangle similarity, 108
- preserves, 138
- primality, 49
- prime, 48
- prime decomposition, 49
- prime factorisation, 49
- Principle of Mathematical Induction, 23
- proof by contradiction, 15
- proof by contraposition, 14
- proper divisor, 48, 64
- proper subset, 10
- Properties of Congruence modulo  $m$ , 58
- Properties of Divides, 48
- proposition, 13
- pseudograph, 161
- punctured

- ball, 147
- neighbourhood, 147
- Pythagoras' Theorem, 109
- $\mathbb{Q}$ 
  - set of Rational Numbers, 8
- QM
  - 2-Power Mean, 96
  - Quadratic Mean, 96
- QM-AM-HM, 96
- quadrangle, 119
  - 4-gon (quadrilateral), 119
- Quadratic Mean
  - QM, 96
- quadratic polynomial, 34
- quadrilateral, 119
  - 4-gon, 119
- quantifiers, 11
- quo
  - computing, 59
- quotient, 33, 47
- $\mathbb{R}$ 
  - set of Real Numbers, 8
- radical axis, 122
- radius, 119
- range
  - of function, 141
- $\text{range}(f)$ 
  - range of function  $f$ , 142
- Ratio Test, 153
- rational, 139
- Rational Numbers
  - $\mathbb{Q}$ , 8
- Rational Zero Theorem, 34
- ray, 105, 119
- Real Numbers
  - $\mathbb{R}$ , 8
- Rearrangement Inequality, 94
- recurrence (relation), 79
- reflex angle, 119
- reflexivity, 19, 20
- regular, 119, 163
  - $r$ -regular, 163
- relatively prime, 55
- remainder, 33, 47
- Remainder Theorem, 33
- removable singularity, 150
- restriction, 142
- rhombus, 119
- RHS Rule
  - triangle congruence, 106
- right angle, 119
- right triangle, 119
- right-angled triangle, 119
- roots
  - of a polynomial equation, 32
- RSA Theorem, 68
- $r$ -sequence, 41
- $r$ -set, 41
- SAS Rule
  - triangle congruence, 106
- SAS, SSS, ASA, RHS Rules, 106
- scalar, 131, 134
- scalar multiplication, 132
- scalar product, 136
- secant, 119
- secondary induction
  - giant version, 27
  - Jake the Peg version, 27
- sector, 119
- sequence, 151
- sequence of partial sums, 152
- series, 151
- Sieve of Eratosthenes, 49
- similar, 108, 119
- simple, 119
- simple graph, 161
- simple induction, 23, 27
  - Aladdin's version, 27
- simson, 118, 119
- Simson line, 119
- Sine Rule, 109
- size, 161
- solutions
  - of a polynomial equation, 32
- spanning tree, 163
- SSS Rule
  - triangle congruence, 106
- Steiner-Lehmus Theorem, 127
- Stewart's Theorem., 125
- straight angle, 105, 119

- strict partial order, 20
- strong induction, 27
- strong trichotomy, 138
- subgraph, 162
- subset, 10
- subtraction, 139
- sufficient condition, 13
- sum of digits of  $N$ 
  - $S(N)$ , 59
- supertransitive, 20
- supplementary angles, 108, 119
- surjective, 144
- symmetry, 19
- synthetic division, 37
  
- take, 10
- tangent, 119
- Tangent-chord Theorem
  - Alternate Segment Theorem, 115
- tangential, 119
- tautology, 11
- term, 77
- terminals, 161
- Theorem
  - Ceva, 120
  - Euler's, 62
- theorem, 13
- total order, 20
- touch, 119
- tournament, 163
- trail, 162
  - $u$ - $v$  trail, 162
- transitivity, 19, 20
  - of ordered field, 138
- transversal, 108, 119
- trapezium, 119
- trapezoid, 119
- tree, 163
- triangle
  - 3-gon, 119
- Triangle Inequality, 90, 140
- Triangle Law
  - for vectors, 132
- trichotomy, 20
- trigon
  - 3-gon (triangle), 119
- trigonometry
  - essential, 110
- trilateral
  - 3-gon (triangle), 119
  
- union, 10
- unique, 137
- unit, 48
- unit vector, 134
- universal quantifier
  - $\forall$ , 11
  
- value, 141
- vector, 131
- vector addition, 132
- vertex, 119, 161
- vertically opposite, 105
- vertices, 161
- Viète's Theorem, 36
  
- w.l.o.g.
  - without loss of generality, 21
- w.r.t.
  - with respect to, 138
- walk, 162
  - $u$ - $v$  walk, 162
  - closed, 162
- weak trichotomy, 138
- Wilson's Theorem, 64
  
- $\mathbb{Z}$ 
  - set of Integers, 8
- zero, 137
  - of a polynomial, 32
- zero polynomial, 31
- zero vector, 134