The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

1994 Senior Mathematics Contest Problems

1. Let $ABCD$ be a parallelogram, M the midpoint of its diagonal BD and P be the point on AD such that $AD = 3PD$.

   If quadrilateral $ABMP$ has area $1995\,\text{cm}^2$, what is the area of $\triangle PDM$?

2. Determine the largest positive integer which, for all positive integers n, is a factor of

$$n^4(n-1)^3(n-2)^2(n-3).$$

3. Let $N \in \mathbb{N}$ with $2n \geq 4$ digits such that the $n-1$ leftmost digits of N are 1s, the next n digits are 2s, and the rightmost digit is 4.

   Prove that N is the product of two positive integers whose digits are all 3s except for the rightmost digit, which is 4 for one of the numbers and 6 for the other.

4. Let $ABCD$ be a cyclic quadrilateral. Let E be the intersection of BA (extended) and CD (extended), and let F be the intersection of CB (extended) and DA (extended). Suppose that A is the incentre of $\triangle CEF$.

   Determine the angles of $\triangle ABD$.

5. Let $f : \mathbb{R} \to \mathbb{R} \setminus \{0\}$ be a function such that

$$f(x+2) = f(x-1)f(x+5),$$

   for all $x \in \mathbb{R}$.

   Prove that f is a periodic function.