The University of Western Australia
DEPARTMENT OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

1995 Senior Mathematics Contest Problems

1. Let $\triangle ABC$ have area 1, let x be such that $0 < x \leq 1$, and let A', B', C' be points on BC, CA, AB, respectively, such that $BA' : A'C = CB' : B'A = AC' : C'B = (1-x) : x$.

   Find the area of $\triangle A'B'C'$ in terms of x.

2. The digits $1234567891011\ldots19941995$ are written on the board forming the number N_1. The digits of N_1 at even places are wiped off the board. Denote the remaining number by N_2. Now the digits of N_2 at odd places are wiped off the board, and the remaining number is denoted by N_3. The digits of N_3 at even places are wiped off the board. Let N_4 denote the number that is left on the board, and so on, continuing the process until only one digit remains on the board.

   Find the last remaining digit.

   Note. The places of digits are counted from the left, e.g. in the number 12345, the digit 1 is in the first place.

3. Determine all (p_1, p_2, p_3, p_4) of primes that satisfy

   (i) $p_1 < p_2 < p_3 < p_4$ and

   (ii) $p_1p_2 + p_2p_3 + p_3p_4 + p_4p_1 = 882$.

4. Determine all polynomials $p(x)$ with real coefficients such that

$$tp(t-1) = (t-2)p(t) \quad \text{for all } t \in \mathbb{R}.$$

5. Let $\triangle$ be a right-angled triangle with the following properties:

   (i) both sides enclosing the right angle are of integer length, and

   (ii) if the perimeter of $\triangle$ is n units, then its area is n square units.

   Determine the side lengths of $\triangle$.