MATHEMATICS OLYMPIAD TRAINING SESSIONS

2002 Senior Mathematics Contest Problems

1. Find all solutions of the following system of equations:

$$\frac{x_1}{x_1+1} = \frac{x_2}{x_2+3} = \frac{x_3}{x_3+5} = \cdots = \frac{x_{1001}}{x_{1001}+2001} \tag{1}$$
$$x_1 + x_2 + \cdots + x_{1001} = 2002 \tag{2}$$

2. Determine all $x, y \in \mathbb{N}$ such that
$$x! + 24 = y^2 \tag{$*$}$$

3. For each pair (k, ℓ) of integers, determine all infinite sequences of integers $a_1, a_2, a_3, \ldots$ in which the sum of every 28 consecutive numbers equals k and the sum of every 15 consecutive numbers equals ℓ.

4. Determine all functions f that have the properties:

   (i) f is defined for all real numbers,

   (ii) $|f(x)| \leqslant 2002 \leqslant \left|\dfrac{xf(y) - yf(x)}{x-y}\right|$ for all x, y with $x \neq y$.

5. For $\triangle ABC$, let D be the midpoint of BC, $\angle BAD = \angle ACB$ and $\angle DAC = 15^\circ$. Determine $\angle ACB$.