The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2003 Senior Mathematics Contest Problems

1. Prove that there does not exist a natural number which, upon transfer of its leftmost digit to the rightmost position, is doubled.

2. Determine all functions f such that:

   (i) $f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$, and

   (ii) $yf(2x) - xf(2y) = 8xy(x^2 - y^2)$ for all $x, y \in \mathbb{R}$.

3. For any three distinct real numbers x, y, z, let

   $$E(x,y,z) = \frac{(|x| + |y| + |z|)^3}{|(x-y)(y-z)(z-x)|}.$$

   Determine the minimum possible value of $E(x, y, z)$.

4. Let S be a set of 2003 points in three-dimensional space such that each of its subsets consisting of 78 points contains at least 2 points that have distance at most 1 from each other.

   Prove that there is a sphere of radius 1 such that at least 27 points of S lie on or inside it.

5. For $\triangle ABC$, let P be the point on BC and Q be the point on AC such that $BP = AB = AQ$. Suppose $\angle ACB = 30^\circ$, and let O and I be the circumcentre and incentre of $\triangle ABC$.

   Prove that

   (a) $PQ = OI$, and

   (b) $PQ \perp OI$ (extended).