The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2006 Senior Mathematics Contest Problems

1. Let D be a point on side BC of triangle ABC. Let K, L be the circumcentres of triangles ABD and ADC, respectively.

   Prove that triangles ABC and AKL are similar.

2. Prove that, among any fifteen composite numbers selected from the first 2006 positive integers, there will be two that are not relatively prime.

3. For each integer n, let a_n be the integer nearest to $\sqrt{n}$.

   Prove that, for each positive integer n, the equation

   $$a_1 + \cdots + a_{n^2+n} = 2(1^2 + \cdots + n^2)$$

   holds.

4. Triangle ABC has a right angle at C. Suppose that D is the point on AB such that CD is perpendicular to AB. Let r_1, r_2 and r be the radii of the incircles of triangles ACD, BCD and ABC, respectively.

   Prove that $r_1 + r_2 + r = CD$.

5. Let $a_3, a_4, \ldots, a_{2005}, a_{2006}$ be real numbers with $a_{2006} \neq 0$.

   Prove that there are not more than 2005 real numbers x such that

   $$1 + x + x^2 + a_3x^3 + a_4x^4 + \cdots + a_{2005}x^{2005} + a_{2006}x^{2006} = 0.$$