The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2007 Senior Mathematics Contest Problems

1. Let $ABCD$ be a trapezium, where sides AB and CD are parallel. Its diagonals AC and BD intersect in E. Further, let F be the point on AD such that EF and AB are parallel.

   Prove that $AF \times BD \times CE = AC \times DF \times BE$.

2. Determine all triples (x, y, p) of integers such that $x^4 + 4y^4 = p$ and p is a prime number.

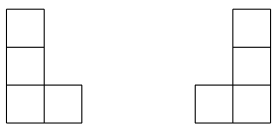
3. For each positive integer n, let $f(n)$ be the smallest of all positive integers k such that n divides $k!$.

   Determine the largest value of $\dfrac{f(n)}{n}$ as n runs over all composite numbers greater than 4.

4. Let ABC and $AB'C'$ be similar right-angled triangles with right angle at C and C', respectively. Let ℓ be the line through C and C', and let D and D' be the points on ℓ such that BD and $B'D'$ are perpendicular to ℓ.

   Prove that $CD = C'D'$.

5. An L-tetromino is a tile in the shape of a letter L, or of its mirror image, consisting of four unit squares as shown below:

   I have six L-tetrominoes of either shape and a 5×5 grid of unit squares. I can place six of these L-tetrominoes in the grid so that there is only one square of the grid not covered.

   Where can that square be?