The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

2009 Senior Mathematics Contest Problems

1. In $\triangle ABC$, let M_a, M_b, M_c be the midpoints of sides BC, CA, AB, respectively, and F_a, F_b, F_c be the feet of the altitudes from A, B, C, respectively.

   Prove that
   $$M_aF_b + M_bF_c + M_cF_a = F_aM_b + F_bM_c + F_cM_a.$$

2. Determine all pairs of real numbers (a_0, a_1) such that

   (i) $a_0 + a_1 = 2008$ and

   (ii) $x^2 + a_1x + a_0 = 0$ has only non-negative solutions.

3. The non-negative real numbers $a_1, a_2, a_3, \ldots$ satisfy the conditions

   (i) $a_n \geq a_{n+1}$ for $n \in \mathbb{N}$ and

   (ii) $a_1 + a_2 + \cdots + a_n \leq 1$ for $n \in \mathbb{N}$.

   Prove that
   $$a_1^2 + 3a_2^2 + 5a_3^2 + \cdots + (2n-1)a_n^2 \leq 1 \text{ for all } n \in \mathbb{N}.$$

4. In acute-angled triangle ABC, let H_a and H_c be the feet of the altitudes through A and C, respectively. In $\triangle AH_aC$, let F_b be the foot of the altitude through H_a and, in $\triangle AH_aB$, let F_c be the foot of the altitude through H_a. Further, let P be the intersection of CH_c with F_bF_c.

   Prove that $H_cF_cH_aP$ is a rectangle.

5. A set S of integers is called *indifferent* if the difference of any two integers in S does not divide the sum of the same two integers.

   What is the largest number of integers that can be chosen from among the numbers $1, 2, 3, \ldots, 2008, 2009$ to form an indifferent set?