AMOC SENIOR CONTEST

Tuesday, 9 August 2016
Time allowed: 4 hours
No calculators are to be used.
Each question is worth seven points.

1. Determine all triples (a, b, c) of distinct integers such that a, b, c are solutions of

$$x^3 + ax^2 + bx + c = 0.$$

2. Each unit square in a 2016×2016 grid contains a positive integer. You play a game on the grid in which the following two types of moves are allowed.

- Choose a row and multiply every number in the row by 2.
- Choose a positive integer, choose a column, and subtract the positive integer from every number in the column.

You win if all of the numbers in the grid are 0. Is it always possible to win after a finite number of moves?

3. Show that in any sequence of six consecutive integers, there is at least one integer x such that

$$(x^2 + 1)(x^4 + 1)(x^6 - 1)$$

is a multiple of 2016.

4. Consider the sequence $a_1, a_2, a_3, \ldots$ defined by $a_1 = 1$ and

$$a_n = n - \lfloor \sqrt{a_{n-1}} \rfloor, \qquad \text{for } n \geq 2.$$

Determine the value of a_{800}.

(Here, $\lfloor x \rfloor$ denotes the largest integer that is less than or equal to x.)

5. Triangle ABC is right-angled at A and satisfies $AB > AC$. The line tangent to the circumcircle of triangle ABC at A intersects the line BC at M. Let D be the point such that M is the midpoint of BD. The line through D that is parallel to AM intersects the circumcircle of triangle ACD again at E.

Prove that A is the incentre of triangle EBM.

© 2016 Australian Mathematics Trust