The University of Western Australia
DEPARTMENT OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Tournament 29 Northern Spring 2008 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. In convex hexagon $ABCDEF$, $AB = DE$ and sides AB, BC, CD are respectively parallel to DE, EF, FA.
   Prove $BC = EF$ and $CD = FA$. (3 marks)

2. There are ten congruent segments on a plane.
   Each point of intersection divides every segment passing through it in the ratio $3 : 4$.
   Find the maximum number of points of intersection. (5 marks)

3. There are ten cards with the number a on each, ten with the number b, and ten with the number c, where a, b and c are distinct real numbers. For every five cards, it is possible to add another five cards so the sum of the numbers on these ten cards is 0.
   Prove that one of a, b and c is 0. (5 marks)

4. Find all $n \in \mathbb{N}$ such that $(n+1)!$ is divisible by $1! + 2! + \cdots + n!$. (6 marks)

5. Each cell of a 10×10 board is painted red, blue or white, with exactly twenty of them red. No two adjacent cells are painted the same colour. A domino consists of two adjacent cells, and is said to be *good* if one of its cells is blue and the other white.

   (a) Prove that it is always possible to cut out 30 good dominoes from such a board. (2 marks)

   (b) Give an example of such a board from which it is possible to cut out 40 good dominoes. (2 marks)

   (c) Give an example of such a board from which it is not possible to cut out more than 30 good dominoes. (2 marks)