INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 42, Northern Spring 2021 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Is it possible that a product of 9 consecutive positive integers is equal to a sum of 9 consecutive (not necessarily the same) positive integers? (4 marks)

2. For $\triangle ABC$, AX and BZ are altitudes, and AY and BT are angle bisectors. Furthermore, angles XAY and ZBT are equal.
Is it necessarily the case that $\triangle ABC$ is isosceles? (4 marks)

3. Maria has a balance scale that can indicate which of its pans is heavier or whether they have equal weight. She also has 4 weights that look the same but have masses of 1001, 1002, 1004 and 1005 g. Can Maria determine the mass of each weight in 4 weighings? The weights for a new weighing may be chosen when the result of the previous ones is known. (4 marks)

4. (a) Is it possible to split a square into 4 isosceles triangles such that no two are congruent? (3 marks)

(b) Is it possible to split an equilateral triangle into 4 isosceles triangles such that no two are congruent? (3 marks)

5. There are several dominoes on a board such that each domino occupies two adjacent cells and no pair of dominoes is adjacent by side or vertex. The bottom left and top right cells of the board are free. A token starts at the bottom left cell and can move to a cell adjacent by side: one step to the right or upwards at each turn.
Is it always possible to move the token from the bottom left to the top right cell without passing through dominoes if the size of the board is

(a) 100×101 cells? (2 marks)

(b) 100×100 cells? (4 marks)