The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2000 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Each of the 16 squares in a 4×4 table contains a number. For any square, the sum of the numbers in the squares sharing a common side with the chosen square is equal to 1. Determine the sum of all 16 numbers in the table. (R Zhenodarov, 3 points)

2. Given that $ABCD$ is a parallelogram, M is the midpoint of side CD and H is the foot of the perpendicular from B to line AM, prove that $\triangle BCH$ is isosceles. (M Volchkevich, 3 points)

3. (a) On a blackboard are written 100 different numbers. Prove that you can choose 8 of them so that their average value is not equal to that of any 9 of the numbers on the blackboard. (2 points)

   (b) On a blackboard are written 100 integers. For any 8 of them, you can find 9 numbers on the blackboard so that the average value of the 8 numbers is equal to that of the 9. Prove that all the numbers on the blackboard are equal. (A Shapovalov, 4 points)

4. Among a set of 32 coins, all identical in appearance, 30 are real and 2 are fake. Any two real coins have the same weight. The fake coins have the same weight, which is different from the weight of a real coin. How can one divide the coins into two groups of equal total weight by using a balance at most 4 times? (A Shapovalov, 5 points)