The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2007 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Black and white checkers are placed on an 8×8 chessboard, with at most one checker on each cell. What is the maximum number of checkers that can be placed such that each row and each column contains twice as many white checkers as black ones? (3 points)

2. Initially, the number 1 and a non-integral number x are written on a blackboard. In each step, we can choose two numbers on the blackboard, not necessarily different, and write their sum or difference on the blackboard. We can also choose a non-zero number on the blackboard and write its reciprocal on the blackboard. Is it possible to write x^2 on the blackboard in a finite number of moves? (4 points)

3. D is the midpoint of the side BC of $\triangle ABC$. E and F are points on CA and AB respectively, such that $BE \perp CA$ and $CF \perp AB$. If DEF is an equilateral triangle, does it follow that $\triangle ABC$ is also equilateral? (4 points)

4. Each cell of a 29×29 table contains one of the integers $1, 2, 3, \ldots, 29$, and each of these integers appears 29 times. The sum of all the numbers above the main diagonal is equal to three times the sum of all the numbers below this diagonal. Determine the number in the central cell of the table. (5 points)

5. The audience chooses two of five cards, numbered from 1 to 5 respectively. The assistant of a magician chooses two of the remaining cards, and asks the audience to take them to the magician, who is in another room. The two cards are presented to the magician in arbitrary order. By an arrangement with the assistant beforehand, the magician is able to deduce which two cards the audience has chosen only form the two cards he receives. Explain how this may be done. (5 points)