The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2008 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Each of ten boxes contains a different number of pencils. No two pencils of the same colour are in any box. Prove that one can choose one pencil from each box so that all chosen pencils are of different colours. (3 points)

2. We are given fifty distinct positive integers of which twenty-five are not greater than 50. The others are greater than 50, but not greater than 100. No two of the given numbers differ by 50. Find the sum of the fifty numbers. (3 points)

3. Let $A_1A_2A_3$ be an acute-angled triangle inscribed in a circle of radius 2. Prove that one can choose points B_1, B_2 and B_3 on the arcs A_1A_2, A_2A_3 and A_3A_1 respectively, such that the values of the area of the hexagon $A_1B_1A_2B_2A_3B_3$ and perimeter of $\triangle A_1A_2A_3$ are equal. (4 points)

4. We are given three distinct positive integers, one of which is the average of the other two. Can the product of all three numbers be equal to the 2008^{th} power of some positive integer? (4 points)

5. Several athletes started running at one end of a straight track at the same time. Their speeds are different, but constant. When they reach the other end of the track, they turn around and run back to the start point. There they turn around again and run back to the other end of the track, and so on. Some time after the start, all athletes meet at the same point. Prove that such an event will happen again. (4 points)