The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2010 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. In a multiplication table, the entry in the i^{th} row and the j^{th} column is the product ij. From an $m \times n$ subtable with both m and n odd, the interior $(m-2) \times (n-2)$ rectangle is removed, leaving behind a frame of width 1. The squares of the frame are painted alternately black and white.

   Prove that the sum of the numbers in the black squares is equal to the sum of the numbers in the white squares. (4 points)

2. In a quadrilateral $ABCD$ with an incircle, $AB = CD$, $BC < AD$ and $BC \parallel AD$.

   Prove that the bisector of $\angle C$ bisects the area of $ABCD$. (4 points)

3. A $1 \times 1 \times 1$ cube is placed on an 8×8 chessboard so that its bottom face coincides with a square of the chessboard. The cube rolls over a bottom edge so that the adjacent face now lands on the chessboard. In this way, the cube rolls around the chessboard, landing on each square at least once.

   Is it possible that a particular face of the cube never lands on the chessboard? (4 points)

4. In a school, more than 90% of the students know both English and German, and more than 90% of the students know both English and French.

   Prove that more than 90% of the students who know both German and French also know English. (4 points)

5. A circle is divided by $2N$ points into $2N$ arcs of length 1. These points are joined in pairs to form N chords. Each chord divides the circle into two arcs, the length of each being an even integer.

   Prove that N is even. (4 points)