The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2011 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. In $\triangle ABC$, P and Q are points on its longest side AB, such that $AQ = AC$ and $BP = BC$. Prove that the circumcentre of $\triangle CPQ$ coincides with the incentre of $\triangle ABC$. (3 points)

2. Several guests at a round table are eating from a basket containing 2011 berries. Going in clockwise direction, each guest has eaten either twice as many berries as or six fewer berries than the next guest.
   Prove that not all the berries have been eaten. (4 points)

3. From a 9×9 chessboard, all 16 unit squares whose row numbers and column numbers are both even have been removed.
   Dissect the punctured board into rectangular pieces, with as few unit squares as possible. (4 points)

4. The vertices of a 33-gon are labelled with the integers from 1 to 33. Each edge is then labelled with the sum of the labels of its two vertices.
   Is it possible for the edge labels to consist of 33 consecutive numbers? (4 points)

5. On a highway, a pedestrian and a cyclist were going in the same direction, while a cart and a car were coming from the opposite direction. All were travelling at different constant speeds. The cyclist caught up with the pedestrian at 10 o'clock. After a time interval, she met the cart, and after another time interval equal to the first, she met the car. After a third time interval, the car met the pedestrian, and after another time interval equal to the third, the car caught up with the cart.
   If the pedestrian met the car at 11 o'clock, when did he meet the cart? (5 points)