The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2012 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Five students have first names Clark, Donald, Jack, Robin and Steve, and family names (in a different order) Clarkson, Donaldson, Jackson, Robinson and Stevenson. It is known that

   Clark is 1 year older than Clarkson,
   Donald is 2 years older than Donaldson,
   Jack is 3 years older than Jackson, and
   Robin is 4 years older than Robinson.

   Who is older, Steve or Stevenson and by how much? (3 points)

2. Let $C(n)$ be the number of prime divisors of $n \in \mathbb{N}$, e.g. $C(10) = 2$, $C(11) = 1$, $C(12) = 2$. Is the number of pairs of positive integers (a, b) such that $a \neq b$ and

   $$C(a + b) = C(a) + C(b)$$

   finite or infinite? (4 points)

3. A 10×10 table is filled out according to the rules of the 'Minesweeper' game: each cell either contains a mine or a number that shows how many mines are in neighbouring cells, where cells are neighbours if they have a common edge or vertex.

   If all mines are removed from the table and then new mines are placed in all previously mine-free cells, with the remaining cells to be filled out with the numbers according to the 'Minesweeper' game rule as above, can the sum of all numbers in the table increase? (5 points)

4. A circle touches sides AB, BC, CD of a parallelogram $ABCD$ at points K, L, M, respectively.

   Prove that the line KL bisects the altitude of $ABCD$ that is dropped to the side AB from C. (5 points)

5. For a class of 20 students several excursions were arranged with at least one student attending each of them.

   Prove that there was an excursion such that each student in that excursion took part in at least 1/20 of all excursions. (5 points)