The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Senior Paper: Years 11, 12
Northern Autumn 2012 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. An $m \times n$ table is filled out according to the rules of the 'Minesweeper' game: each cell either contains a mine or a number that shows how many mines are in neighbouring cells, where cells are neighbours if they have a common edge or vertex.

   If all mines are removed from the table and then new mines are placed in all previously mine-free cells, with the remaining cells to be filled out with the numbers according to the 'Minesweeper' game rule as above, can the sum of all numbers in the table increase? (4 points)

2. We are given a convex polyhedron and a sphere that intersects each edge of the polyhedron in two points such that each edge is split into 3 equal parts.

   Is it necessarily true that all faces of the polyhedron are

   (a) congruent polygons? (2 points)

   (b) regular polygons? (3 points)

3. For a class of 20 students several excursions were arranged with at least four students attending each of them.

   Prove that there was an excursion such that each student in that excursion took part in at least 1/17 of all excursions. (5 points)

4. Let $C(n)$ be the number of prime divisors of $n \in \mathbb{N}$.

   (a) Is the number of pairs of positive integers (a, b) such that $a \neq b$ and

   $$C(a+b) = C(a) + C(b)$$

   finite or infinite? (2 points)

   (b) Answer the above question, if also $C(a+b) > 1000$. (3 points)

5. There are two identical fake coins amongst 239 coins of similar appearance. A fake coin has a different mass to a genuine coin.

   Determine in three weighings (on a balance without weights) whether the fake coins are heavier or lighter than the genuine ones. (5 points)