The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Spring 2010 (A Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. There is a piece of cheese. It is possible to choose any positive number (not necessarily an integer) $a \neq 1$ and cut up the piece of cheese in the ratio $1 : a$ according to its mass. Then, one can keep cutting up any resulting pieces, in the same ratio any of the existing pieces.

   Is it possible to follow this process in such a way that all the resulting pieces of cheese could be put into two piles of equal mass after a finite number of cuttings? (3 points)

2. Let ABC be a triangle in which M is the midpoint of AC and point P lies on BC, AP meets BM at O, and $BO = BP$.

   Find the ratio $OM : PC$. (4 points)

3. There are 999 numbers located on the circumference of a circle. Each of them is equal to either 1 or -1, and both possible numbers occur. Someone computes all products of 10 consecutive numbers and adds them.

   (a) What is the least sum which can be obtained? (3 points)

   (b) What is the greatest sum which can be obtained? (3 points)

4. The sum of the digits of a positive integer n is equal to 100.

   Is it possible that the sum of the digits of n^3 is equal to 100^3? (6 points)

5. (a) Three knights are riding along a circular road in an anti-clockwise direction. There is a single point on the road where a knight may overtake another.

   Is it possible that they can ride for an arbitrarily long period of time with pairwise distinct constant speeds? (3 points)

   (b) What is the answer if there are ten knights instead? (5 points)

6. A polygonal curve in the plane is not closed, has no self-intersections and consists of 31 edges (neighbouring segments do not lie on the same straight line). Someone has drawn all lines containing these edges. This results in 31 lines (some of which may be coincident).

   What is the minimum possible number of distinct lines? (8 points)

7. There are fleas in some squares of a 10×10 chessboard (no more than a single flea in a square). Once a minute each flea jumps from its square to an adjacent one (that is, to a square with which it has a common side). All fleas jump simultaneously. Each flea keeps the same direction of jumping unless on the next jump it would leave the board, in which case it changes to the opposite direction. During a whole hour, there was not a minute when any square was occupied by two or more fleas.

   Find the maximum possible number of fleas on the board. (11 points)