The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

**AMO/TT TRAINING SESSIONS**

# Tournament of the Towns Problems
# Junior Paper: Years 8, 9, 10
# Northern Spring 2011 (A Level)

**Note:** Each contestant is credited with the largest sum of points obtained for three problems.

1. Does there exist a hexagon that can be divided into four congruent triangles by a straight cut? (4 points)

2. Passing through the origin of the coordinate plane are 180 lines, including the coordinate axes, each forming $1^\circ$ angles with the next.

   Determine the total sum of the $x$-coordinates of the points of intersections of these lines with the line $y = 100 - x$. (4 points)

3. Baron Münchausen has a set of 50 coins. The masses of the coins are distinct positive integers, not exceeding 100, and their total mass is even. The Baron claims that it is not possible to divide the coins into two piles with equal total mass.

   Can the Baron be right? (5 points)

4. Let $n \in \mathbb{N}$.

   Prove that there exist two pairs of positive integers such that the sums of the numbers in both pairs are equal while one corresponding product equals $n$ times the other. (6 points)

5. In $\triangle ABC$, $AD$ and $BE$ are altitudes. From $D$, perpendiculars are dropped to $AB$ at $G$, and $AC$ at $K$. From $E$, perpendiculars are dropped to $AB$ at $F$ and $BC$ at $H$.

   Prove that $FGHK$ is an isosceles trapezium. (7 points)

6. Two ants crawl along the edges of the squares of a $7 \times 7$ board. Each ant passes through all 64 vertices exactly once and returns to its starting point.

   What is the smallest possible number of edges covered by both ants? (10 points)

7. In every cell of a square table is a number. The sum of the largest two numbers in each row is $a$ and the sum of the largest two numbers in each column is $b$.

   Prove that $a = b$. (10 points)