The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Spring 2009 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. All diagonals are drawn in a convex 2009-gon. A straight line intersects a 2009-gon in such a way that no vertex of the polygon lies on the line. Prove that the line intersects an even number of the polygon diagonals. (3 points)

2. Let $a \wedge b$ denote the number a^b. It is required to arrange brackets in the expression $7 \wedge 7 \wedge 7 \wedge 7 \wedge 7 \wedge 7 \wedge 7$ to identify the order of operations (in total 5 pairs of brackets). Is it possible to make two different bracket arrangements which give the same value? (4 points)

3. Volodya wants to have a set of equal size cubes with one digit written on each face of each cube, so that any 30-digit number can be composed from the cubes. What is the least number of cubes required for him to do that? (Digits 6 and 9 cannot be converted one into another by a cube's rotation.) (4 points)

4. A positive integer increased by 10% is also an integer. Is it possible that the sum of digits of the number can be decreased exactly by 10% by this process? (4 points)

5. Let $ABCD$ be a rhombus with $\angle A = 120^\circ$. Points M and N lie on sides BC and CD, respectively, such that $\angle NAM = 30^\circ$. Prove that circumcentre of $\triangle NAM$ lies on a diagonal of the rhombus. (5 points)