The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Spring 2010 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. There are some pears, plums and apples in six baskets. The number of plums in each basket is equal to the number of apples in all other baskets together, while the number of apples in each basket is equal to the number of pears in all other baskets together.

   Prove that the total number of items of fruit is divisible by 31. (3 points)

2. Lillebror and Karlsson have a square cake to cut up. Karlsson chooses a point at the cake (not on an edge). Next Lillebror makes a straight cut from that point to an edge (in any direction). Then Karlsson makes another straight cut from the point he first chose, to the edge of the cake, perpendicular to Lillebror's cut. The smaller of two pieces goes to Lillebror. Lillebror wants to have at least a quarter of the cake. Can Karlsson prevent him from having such a piece of cake? (3 points)

3. An angle is drawn and a compass is the only available tool.

   (a) What is the smallest number of circles which must be constructed to determine accurately whether the given angle is acute? (2 points)

   (b) How could you determine whether the given angle is equal to 31°? (It is allowed to construct as many circles as needed.) (2 points)

4. Each Olympiad participant has acquaintance with at least three other participants. Prove that it is possible to choose a group with an even number of more than two participants, that can be arranged around a circular table in such a way that each participant has acquaintance with both his/her neighbours. (5 points)

5. There are 101 numbers written on a board: $1^2, 2^2, \ldots, 101^2$. At each step, two numbers are erased and replaced by (the absolute value of) their difference. What is the smallest number that can be obtained as the result of 100 steps? (5 points)