The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Spring 2011 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. The numbers from 1 to 2010 inclusive are placed around a circle so that as one moves clockwise around the circle, the numbers increase and decrease alternately.

   Prove that the difference of two adjacent integers is even. (3 points)

2. A rectangle is divided into 121 rectangular cells by 10 vertical and 10 horizontal lines so that 111 cells have integer perimeters.

   Prove that the remaining ten cells also have integer perimeters. (4 points)

3. Worms grow at the rate of 1 metre per hour. When they reach their maximum length of 1 metre, they stop growing. If a worm is fully grown, one can dissect it at any point along its length, into two parts, so that two new worms arise, which, since their lengths are now less than 1 metre in length, grow at the rate of 1 metre per hour.

   Starting with 1 fully grown worm, can one obtain 10 fully grown worms in less than 1 hour? (5 points)

4. Each diagonal of a convex quadrilateral divides it into two isosceles triangles. The two diagonals of the same quadrilateral divide it into four isosceles triangles.

   Must this quadrilateral be a square? (5 points)

5. A dragon has imprisoned a knight and given him 100 distinct coins, half of which are magic (only the dragon knows which coins are magic). Every day the knight splits all coins into two piles (not necessarily equal). If two piles include either an equal number of magic coins or an equal number of ordinary coins, then the dragon will release the knight.

   Can the knight guarantee himself freedom in at most

   (a) 50 days? (2 points)

   (b) 25 days? (3 points)