The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Senior Paper: Years 11, 12
Northern Spring 2010 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. 2010 ships loaded with bananas, lemons and pineapples travel from South America to Russia. The number of bananas on each ship is equal to the number of lemons on all other ships combined, while the number of lemons on each ship is equal to the number of pineapples on all other ships combined.

   Prove that the total number of items of fruit is divisible by 31. (3 points)

2. The following is known about a function $f(x)$:

   > Any straight line in the x-y plane has the same number of intersections with the graph of $y = f(x)$ as with the graph of the parabola $y = x^2$.

   Prove that $f(x)$ is equal to x^2. (4 points)

3. Is it possible to glue some regular hexagons over the surface of a regular octahedron, without overlaps or gaps?

   Note. A regular octahedron has 6 vertices, all of its faces are equilateral triangles, and each vertex is incident with 4 faces. (5 points)

4. Baron Munchausen asks that a non-constant polynomial $P(x)$ with non-negative integer coefficients be chosen, and asks to be told only the values $P(2)$ and $P\big(P(2)\big)$. The Baron claims that he will be able to identify the polynomial $P(x)$ given only these two values.

   Isn't the Baron mistaken? (5 points)

5. There is a needle lying in a plane. One is allowed to rotate the needle 45° around either of its ends.

   Is it possible, after some rotations, for the needle to end up in its initial position, but with its ends swapped?

   Note. Consider a needle to be a line segment. (6 points)