

AMO TRAINING SESSIONS

**Tournament of the Towns Questions**

1. Find all real solutions to the system of equations

$$(x + y)^3 = z, \quad (y + z)^3 = x, \quad (z + x)^3 = y.$$

(Based on an idea by A. Aho, J. Hopcroft, J. Ullman, 5 points)

2. For every natural number  $n$  prove that

$$\begin{aligned} & \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)^2 + \left(\frac{1}{2} + \cdots + \frac{1}{n}\right)^2 + \cdots + \left(\frac{1}{n-1} + \frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)^2 \\ & = 2n - \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right) \end{aligned}$$

(S. Manukian, Yerevan, 4 points)

3. When  $4444^{4444}$  is written in decimal notation, the sum of its digits is  $A$ . Let  $B$  be the sum of the digits of  $A$ . Find the sum of the digits of  $B$ .