Olympiad News

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Since the last column there have two Olympiad events: the Western Australian Junior Mathematics Olympiad (WAJO), and the Tournament of the Towns (TT, Northern Autumn round).

The 2011 WAJO was held on the first Saturday of November. This is a fast and furious competition, targeted at the Year 9 level, with an individual paper of 10 questions worth 25 marks to be completed in 100 minutes, and a team paper worth 45 marks to be completed in 45 minutes. Students are organised (mostly) into teams of four, with a team's score being the sum of the four students' individual paper scores together with their team paper score, for a total possible score of 145. If a team only has three students, their individual scores are scaled accordingly. The idea is that the teams are made up of students from the same school, so that they represent their school. Where this is not possible, so-called "Allies" teams are formed. The papers are marked on the day, and while students have a break, after the team competition, marks are finalised, and then the Awards Ceremony of the competition is held, with Individual and Team prizes for Year 9 and Year 8. All in all, it's a lot of fun for everyone involved.

The 2011 WAJO set new records, with 378 students participating, 50 more than the 2009 record. These students were in 97 teams (16 more than the 2009 record), and from 44 schools, which curiously was one fewer than in 2010. Full details of the event are available at the website:

http://enrichmaths.sponsored.uwa.edu.au/home/wajo

The side menu link **2011 Olympiad**, gives a page with the full list of prizes awarded together hopefully with links to photos, by the time this article reaches press. As was the case last year, four students achieved a perfect score in the Individual competition: Tristan Taylor (Frederick Irwin Anglican School), Jonathan Pang (Hale School), Brendon Wright (Christ Church Grammar School), all Year 9 students, and Hyeonkyoo Yoo (Perth Modern School) of Year 8. Photos of the students in action, provided by Monique Ellement, will be found by following links from the **Olympiad Stats** page, which also provides summary statistics for all previous Olympiads, and **Past Questions and Solutions** where each year's WAJO's questions are provided by year, complete with all solutions. There is also an **Announcement** of the next WAJO, which is set for 3 November, 2012. In 2011, UniDiscovery and Aspire UWA joined the many sponsors who provide prizes or otherwise assist in running the event. The WA Minister of Education, Dr Elizabeth Constable, honoured us again by presenting the prizes sponsored by the Ministry of Education, personally and staying for the full Awards Ceremony.

Also, worthy of special mention are three special prizes, namely the **Phill Schultz Prize** and two **Special WAMOC Awards,** that though not part of the WAJO itself are awarded at the WAJO Awards Ceremony. The Phill Schultz Prize is awarded to the high school student who, in the opinion of the WA Mathematical Olympiads Committee has demonstrated the most outstanding performance in Mathematics Challenge activities such as Mathematical Olympiads and other competitions during the previous year. In 2011, the prize was awarded (again) to Angel Yu, a Year 12 student from Perth Modern School, whose 2011 achievements were:

2011	Australian Mathematics Olympiad (Gold Certificate) - 1st in Australia!
2010-2011	Tournament of the Towns (Diploma - Senior Division)
2011	International Mathematics Olympiad (Bronze Medal)

What is especially significant about the first and last of the above achievements, is that the last time a Western Australian either obtained an AMO Gold Certificate or was a member of Australia's IMO team was in 2001. However, Angel was not the only WA student to receive a Gold Certificate in the AMO this year. As was reported earlier in the year, Alexander Chua (Year 10, Christ Church Grammar School), Xin Zheng Tan (Year 12, All Saints' College) and Andrew Yang (Year 11, Rossmoyne Senior High School) also achieved Gold in the AMO. Alexander Chua followed up with perfect scores in the Senior Mathematics Contest (SMC) and Australian Intermediate Mathematics Olympiad (AIMO), which earned him an invitation to the December School of Excellence as a Senior (which as reported in previous columns, is a training school for the IMO), and was consequently awarded one of the two 2011 Special WAMOC Awards.

The Special WAMOC Awards have been awarded since 2007, to support WA students who have been invited to the School of Excellence, and up until 2010, only to those who were invited to the December School. However, Andrew Yang's extraordinary achievement of a Gold certificate in the AMO earned him an invitation to the School of Excellence in April. The WA Mathematics Olympiad Committee felt this was equally deserving of a Special WAMOC Award. So, in summary the students who received a 2011 Special WAMOC Award, with what earned them an invitation to a School of Excellence, were:

Alexander Chua	(Year 10)	35 (Prize) SMC, 35 (Prize) AIMO
Andrew Yang	(Year 11)	Gold Certificate in AMO

We don't expect to have to wait another ten years before Western Australia will again have a representative in Australia's IMO team, since we hope that Alexander Chua will be a member of Australia's next two IMO teams!

TT, Northern Autumn round, for 2011, was held on Saturday, 26 November (O Level paper) and Saturday, 3 December (A level paper). The Tournament of the Towns is an "invitation-only" maths competition; a first invitation for students was made on the basis of a strong AIMO result or a significant WAJO achievement. The O paper is a 4-hour paper with five questions, and the A paper is a 5-hour paper with seven questions. A student's score on a paper is the highest total for their attempts at three of the questions. A student's overall score for the TT round is the higher score of the two papers. Twelve students (ten juniors and two seniors) gained Distinctions and have had their papers forwarded to Moscow for a more rigorous marking; and hopefully they will receive a Diploma from the Russian Academy of Sciences, to go with their certificate from the Australian Mathematics Trust. A summary of the results in order of rank is below.

Junior Student	*Year*	*School*	*Result*	*WA Rank*
Alexander Chua	10	Christ Church GS	Distinction	1
Nicholas Pizzino	7	Christ Church GS	Distinction	2
Hyeon Kyoo (Henry) Yoo	8	Perth Modern School	Distinction	3
Nicholas Lim	9	Christ Church GS	Distinction	=4
Zhixian Wu	9	Perth Modern School	Distinction	=4
Satthya Krishnasavim	9	Perth Modern School	Distinction	=6
Conway Li	10	Perth Modern School	Distinction	=6
Diffy Zhou	10	Perth Modern School	Distinction	=6
Edward Yoo	10	All Saints' College	Distinction	9
Albert Qiu	8	Christ Church GS	Distinction	10
Daryl Chung-Wah-Cheong	10	Perth Modern School	Credit	11
Mingzhao Liu	10	Rossmoyne SHS	Participation	12
Vandit Trivedi	10	Christ Church GS	Participation	13
Ciaran Murray	10	Trinity College	Participation	=14
Joseph Thompson	10	Perth Modern School	Participation	=14
Christopher Yeung	10	Wesley College	Participation	16

Senior Student	*Year*	*School*	*Result*	*WA Rank*
Andrew Yang	11	Rossmoyne SHS	Distinction	1
Aaron Hurst	11	Home School	Distinction	2
Aaron Wong	12	Aquinas College	Credit	3

While we're here, perhaps we should mention that Calum Braham (Trinity College), one of the 2011 Beazley Medal winners (congratulations Calum!) featured in last year's list of TT Distinctions.

The next Olympiad event is the Australian Mathematics Olympiad (AMO) which will be held on Tuesday 14 and Wednesday 15 February. There will be a report on the results of the 2012 AMO in the next column. Finally, let us close with a problem from each of the two Olympiads featured in this column.

Question 6 (WAJO):

How many pairs of integers (x,y) satisfy $1 \le x/2+1 \le y \le 6$?

Solution. Firstly, the conditions imply $1 \le y \le 6$. Also,

$$1 \le x/2+1 \le y$$
$$0 \le x/2 \le y-1$$
$$0 \le x \le 2(y-1).$$

So, if $y=n$ then x can be any of the integer values $0,1,\dots,2(n-1)$, i.e. $2n-1$ possibilities. As y ranges over the values $1,2,\dots,6$ the numbers of possibilities for x are $1,3,\dots,11$. Hence the number of possible pairs (x,y) is

$$1+3+\cdots+11=(1+11)\times 6/2=36,$$

since the left hand side expression is an arithmetic series with first term 1, last term 11, and 6 terms (and the common difference is 2).

Question 3 (TT, Northern Autumn 2011, Junior A Level)**:**

A set of at least two objects with different masses has the property that for any pair of objects their mass is equal to the mass of a subset of the remaining objects.

What is the minimum number of objects in the set?

Solution. Let n be the minimum number of masses. Firstly, any pair of masses has mass greater than 0, so that there must be at least one more object. Hence, $n \ge 3$.

Suppose $n=3$. Without loss of generality, let the masses be x,y,z with $x<y<z$. Then taking the pair y,z the property implies $x=y+z$. But this is impossible since $x<y<z$ implies $x<y+z$. So $n>3$.

Suppose $n=4$. Without loss of generality, let the masses be w,x,y,z with $w<x<y<z$. Taking the pair y,z we observe that their mass is larger than the remaining objects, i.e. $y+z>w+x$, and so again the property cannot be satisfied. So $n>4$.

Suppose $n=5$. Without loss of generality, let the masses be v,w,x,y,z with $v<w<x<y<z$. Now,

$$y+z>w+x>v+x>v+w$$

so we must have $y+z=v+w+x$. Since $z+x>y+w$, we must also have

$$z+x=y+w+v>x+w+v=y+z,$$

which implies $x>y$ (contradiction). So $n>5$.

On the other hand with the six masses: $3,4,5,6,7,8$ we have:

$$8+7=6+5+4, 8+6=7+4+3, 8+5=7+6, 8+4=7+5, 8+3=7+4=6+5,$$
$$7+3=6+4, 6+3=5+4, 5+3=8, 4+3=7,$$

which shows that the property is satisfied.

Hence the minimum number n of masses is 6.

Remark. If we had tried the masses $1,2,3,4,5,6$ we would have found the property is not satisfied, and might have started looking for a proof that $n > 6$. So you can see this is quite a challenging problem!