Olympiad News

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The last column was prepared just before this year's Australian Mathematics Olympiad (AMO) and just missed reporting on it by a few days. The 2011 AMO – two papers – was held on 8 and 9 February, and the best-performing of the students sitting the AMO were invited to sit the Asia Pacific Mathematics Olympiad (APMO) – a single paper – on 8 March, 2011, each of the papers being of 4 hours in duration.

The results in the AMO, this time round, were particularly significant: 4 WA students achieved Gold! This was one more than New South Wales and two more than Victoria! Also, two more WA students were awarded a Silver certificate, and three WA students received a Bronze certificate. This is the best WA result, ever! The AMO results of those students, and APMO results of the five who were subsequently invited to sit the APMO are listed below.

WA Student	*Year*	*School*	*AMO Result*	*National Rank*
Angel Yu	12	Perth Modern School	Gold	1
Alexander Chua	10	Christ Church GS	Gold	8
Xin Zheng Tan	12	All Saints' College	Gold	9
Andrew Yang	11	Rossmoyne SHS	Gold	12
Edward Yoo	10	All Saints' College	Silver	13
Aaron Wong	12	Aquinas College	Silver	30
Li Kho	12	Willetton SHS	Bronze	=36
Kathleen Dyer	11	St Hilda's ASG	Bronze	=57
Michael Warton	11	Hale School	Bronze	=57

WA Student	*Year*	*School*	*APMO Result*	*National Rank*
Angel Yu	12	Perth Modern School	15 (top was 20)	=6
Alexander Chua	10	Christ Church GS	15	=6
Edward Yoo	10	All Saints' College	8	=15
Xin Zheng Tan	12	All Saints' College	5	=17
Andrew Yang	11	Rossmoyne SHS	5	=17

The full results can be found at:
http://www.amt.edu.au/amo2011.html and http://www.amt.edu.au/apmo2011.html
for AMO, and APMO, respectively.

Particularly outstanding is the student on the top of both of the above tables: Angel Yu. The last time a WA student stood at the top of the AMO honours table, was Peter McNamara in 2001, who represented Australia at the 2000 and 2001 International Mathematical Olympiads (IMO). So this brings up an important question: What does it take to get into the Australian IMO team, and who of the above qualifies this year? The usual preparation required for the IMO is attendance at the Australian Mathematics Trust School of Excellence both as a Junior and a Senior, and generally twice at each level. Of the above, only Angel Yu and Alexander Chua have achieved this, and from the above results both are strongly in contention. The IMO team will be announced on 2 June! The result will have to wait until the next column!

Of the remaining students, Xin Zheng Tan (in December 2009 and April 2010), Kathleen Dyer (in December 2010) and Edward Yoo (in December 2010 and April 2011) have attended the School of Excellence as Juniors. Since Xin Zheng Tan is in Year 12 already, unfortunately he has missed his chance at an IMO, but as you can see, not by much! Notice, Andrew Yang is also in the mix there. Now, Andrew has done something rather special. On the strength of a High Distinction in the Australian Intermediate Mathematics Olympiad last year he was invited to do the Tournament of the Towns at the end of 2010, and on the strength of his result there he was invited to sit the AMO, and achieved Gold! This really is quite amazing, and amazing enough that the convenors of the School of Excellence invited him to the 2011 April School as a Junior. So now there are three students, who if they manage to get an invitation in December 2011 to the School of Excellence as Seniors, who along with Alexander Chua (who will then be in Year 11), will be contenders for the Australian IMO team in 2012. What will these students have to do to get such an invitation as Seniors? Essentially, they will need a High Distinction (or very close to one) in the Senior Contest in August. We will pretty much expect that of Alexander, but the other three still have a very hard road. I'm not going to say we wish them luck, because it's not luck that will get them there; it will be more of what has got them to where they are now: persistence, patience, a certain amount of self-belief, a drive to succeed and sheer dedicated hard work. My hat is off to them!

Also invited to sit the AMO in WA were Jonathan Chung-Wah-Cheong and Calum Braham of Trinity College, and Bojana Surla of Penrhos College, all Year 12 students. Their results were good too, just not of the calibre of the above.

The next Olympiad event is the next round of the Tournament of the Towns, to be held on 6 and 13 May, and will be reported on next time.

Finally, let us close with a question from AMO 2011:

Question 6 (AMO 2011):
Determine all real numbers r such that the three solutions of the equation

$$x^3 - 30x^2 + rx - 780 = 0$$

are the side lengths of a right-angled triangle.

Solution. Let α, β, γ be the three solutions of the equation, and without loss of generality assume $\alpha \le \beta \le \gamma$. Then comparing the left hand side coefficients of the equation with the expansion of

$$(x-\alpha)(x-\beta)(x-\gamma),$$

i.e. with

$$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\alpha\gamma+\beta\gamma)x - \alpha\beta\gamma,$$

we have, along with an application of Pythagoras' Theorem to α, β, γ (since they are side lengths of a right-angled triangle):

$$\begin{array}{ll} (1) & \alpha+\beta+\gamma = 30 \\ (2) & \alpha\beta+\alpha\gamma+\beta\gamma = r \\ (3) & \alpha\beta\gamma = 780 \\ (4) & \alpha^2+\beta^2 = \gamma^2 \end{array}$$

Rearranging (1) and (3) we have

$$\alpha+\beta = 30-\gamma$$

$$\alpha\beta = \frac{780}{\gamma},$$

and using these to eliminate γ in (3), we have

$$\begin{aligned} r &= \alpha\beta + (\alpha+\beta)\gamma \\ &= \frac{780}{\gamma} + (30-\gamma)\gamma. \end{aligned}$$

Now, we use (4), and our expressions for $\alpha+\beta$ and $\alpha\beta$:

$$\begin{aligned}\gamma^2 &= \alpha^2+\beta^2 \\ &= (\alpha+\beta)^2 - 2\alpha\beta \\ &= (30-\gamma)^2 - 2\cdot\frac{780}{\gamma} \\ &= 30^2 - 2\cdot 30\gamma + \gamma^2 - 2\cdot\frac{780}{\gamma} \\ 0 &= 30^2 - 2\cdot 30\gamma - 2\cdot\frac{780}{\gamma} \\ 0 &= 15\gamma - \gamma^2 - 26 \\ 0 &= \gamma^2 - 15\gamma + 26 \\ &= (\gamma-2)(\gamma-13)\end{aligned}$$

So $\gamma = 2$ or $\gamma = 13$. Now, $\gamma = 2$ implies $\alpha, \beta \le 2$ whence

$$\alpha+\beta+\gamma \le 6,$$

contradicting (1). So $\gamma \ne 2$.

On the other hand, 13 reminds us of the Pythagorean triad 5, 12, 13, and a moment's checking of (1), (3), (4), shows that indeed, $\alpha = 2, \beta = 5, \gamma = 13$ works. Thus finally, we have that there is but a single solution for r, namely

$$\begin{aligned} r &= \alpha\beta + (\alpha+\beta)\gamma \\ &= 5\cdot 12 + 17\cdot 13 \\ &= 281.\end{aligned}$$