Olympiad News

Greg Gamble
WA State Director of Mathematics Olympiads
G.Gamble@curtin.edu.au

The first column of the year misses being able to report on the Australian Mathematics Olympiad (AMO) by a few days, and this column will miss reporting on the Tournament of the Towns (the O Level of which, was held a day ago, at time of writing). The 2012 AMO – two papers – was held on 14 and 15 February, and the best-performing of the students sitting the AMO were invited to sit the Asia Pacific Mathematics Olympiad (APMO) – a single paper – on 13 March, 2012, each of the papers being of 4 hours in duration.

The results in the AMO, this time round, were exceptional, though not as spectacular as the 4 Golds, 2 Silver and 3 Bronze of last year. This year, 4 WA students achieved Silver, one achieved Bronze, and one achieved an Honourable Mention (which means full marks for at least one question). Now it would seem that the two top WA students have slipped back a bit, but, in fact, their actual scores are higher than last year, yet nevertheless their certificates are of a lower colour. Even, Edward Yoo, who had the top Silver last year, scored slightly higher this year. But the really big improver was Kathleen Dyer, from just making Bronze last year to top twenty and Silver this year. Well done, Katie! Aaron Hurst (Year 12, Home School) and Jack Cooper (Year 11, Hale School) scored slightly higher than Conway Li, listed below, and Diffy Zhou (Year 11, Perth Modern School) and Ciaran Murray (Year 11, Trinity College) scored slightly lower. The most significant AMO and APMO results of WA students, are listed below. Also sitting the APMO from WA were Andrew Yang and Edward Yoo, who finished just outside the top ten.

WA Student	*Year*	*School*	*AMO Result*	*Aus.National Rank*
Alexander Chua	11	Christ Church GS	Silver	10
Andrew Yang	12	Rossmoyne SHS	Silver	15
Edward Yoo	11	All Saints' College	Silver	16
Kathleen Dyer	12	St Hilda's ASG	Silver	18
Daryl Chung	11	Perth Modern School	Bronze	=36
Conway Li	11	Perth Modern School	Hon. Mention	

WA Student	*Year*	*School*	*APMO Result*	*National Rank*
Alexander Chua	11	Christ Church GS	17	3

Fuller results (but not the results below Bronze) can be found at:
http://www.amt.edu.au/amo2012.html and
http://www.amt.edu.au/apmo2012.html
for AMO, and APMO, respectively. Note that the AMO list includes several New Zealand students, and the Rank above counts by omitting these students.

Edward Yoo's results in the AMO and APMO were deemed high enough to warrant an invitation to the April School of Excellence as a Senior. (Edward was previously invited as a Junior in December 2010 and April 2011). The 2012 International Mathematics Olympiad (IMO), will be held in July, in Mar del Plata, Argentina, and the Australian IMO Team will be announced early June. So the selection result will have to wait until the next column. As mentioned last year, the usual preparation required for the IMO is attendance at the Australian Mathematics Trust School of Excellence both as a Junior and a Senior, and generally twice at each level. Of the above, only Alexander Chua has achieved this for this year, and we hope that at this time next year, Edward Yoo, along with Alexander Chua, will be strongly in contention.

As mentioned above, the subject of the next Olympiad News will be the next round of the Tournament of the Towns, being held on 5 and 12 May.

Finally, let us close with a question from AMO 2012:

Question 1 (AMO 2012):
Determine the largest positive integer n such that $4^n + 2^{2012} + 1$ is a perfect square.

Solution. Observe that $4^n = 2^{2n} = (2^n)^2$ is a perfect square and the next largest square is $(2^n + 1)^2$. Therefore,

$$4^n + 2^{2012} + 1 \geq (2^n + 1)^2 \ldots\ldots\ldots\ldots (*)$$
$$= 2^{2n} + 2 \cdot 2^n + 1$$
$$\Rightarrow 2^{2012} \geq 2^{n+1}$$
$$\Rightarrow 2012 \geq n + 1, \text{since } \log_2(x) \text{ is an increasing function}$$

Hence,

$$n \leq 2011.$$

On the other hand, for $n = 2001$, we have equality at $(*)$, so that $4^{2011} + 2^{2012} + 1$ is a square.
Therefore, $n = 2001$ is the largest positive integer n such that that $4^n + 2^{2012} + 1$ is a perfect square.

Remark. Just in case, the key idea went by you, what we did above was to first show n was bounded above by 2011 and then show that that bound worked. This sort of two-pronged attack is very typical of the techniques needed to solve AMO problems.