Olympiad News

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Since the last column there have two Olympiad events: the Western Australian Junior Mathematics Olympiad (WAJO), and the Tournament of the Towns (TT, Northern Autumn round).

The WAJO, is normally held on the last Saturday of October, and sometimes the first Saturday of November (in 2010, it was 30 October). This is a fast and furious competition, targeted at the Year 9 level, with an individual paper of 10 questions worth 25 marks to be completed in 100 minutes, and a team paper worth 45 marks to be completed in 45 minutes. Students are organised (mostly) into teams of four, with a team's score being the sum of the four students' individual paper scores together with their team paper score, for a total possible score of 145. If a team only has three students, their individual scores are scaled accordingly. The idea is that the teams are made up of students from the same school, so that they represent their school. Where this is not possible, so-called "Allies" teams are formed. The papers are marked on the day, and while students are having lunch, after the team competition, marks are finalised, and then the Awards Ceremony of the competition is held, with Individual and Team prizes for Year 9 and Year 8. All in all, it's a lot of fun for everyone involved.

The 2010 WAJO was 3 students shy of matching the record established in 2009, with 325 students participating. Full details of the event are available at the website:

http://enrichmaths.sponsored.uwa.edu.au/home/wajo

The side menu link **2010 Olympiad**, gives a page with the full list of prizes awarded together with links to photos. Deserving of a special mention are the four students who achieved a perfect score in the Individual competition: Liam Kearney (Christ Church Grammar School), Katerina Chua (St. Hilda's Anglican School for Girls), Shilpa Rath (St. Mary's Anglican Girls' School), all Year 9 students, and Samuel Alsop (Frederick Irwin Anglican School) of Year 8. Photos of the students in action, provided by Monique Ellement, are found by following links from the **Olympiad Stats** page, which provides summary statistics for all previous Olympiads as well as the 2010 WAJO, and **Past Questions and Solutions** where each year's WAJO's questions are provided by year, complete with all solutions. There is also an **Announcement** of the next WAJO, which is set for 5 November 2011, and will start time an hour and a half later than in 2010, in order to better accommodate Country schools attending. The 2010 WAJO saw in some new sponsored prizes from the AISWA (Association of Independent Schools of Western Australia Inc.) and the PFFWA (Parents and Friends' Federation of WA Inc.) and the Education Department prizes were replaced by a number of Minister of Education Awards, for which the WA Minister of Education, Dr Elizabeth Constable, honoured us by presenting the prizes personally and staying for the full Awards Ceremony.

Also, worthy of special mention are four special prizes, namely the **Phill Schultz Prize** and three **Special WAMOC Awards,** that though not part of the WAJO itself are awarded at the WAJO Awards Ceremony. The Phill Schultz Prize is awarded to the high school student who, in the opinion of the WA Mathematical Olympiads Committee has demonstrated the most outstanding performance in Mathematics Challenge activities such as Mathematical Olympiads and other competitions during the previous year. In 2010, the prize was awarded to Angel Yu, a Year 11 student from Perth Modern School, whose most significant achievements were:

2009, 2010	Australian Mathematics Olympiad (Silver Medal) - 1st in WA
2009-2010	Tournament of the Towns (Diploma - Senior Division)
2010	Australian Mathematics Medal - Senior Division) - 1st in WA
2010	Senior Mathematics Contest (Prize) - 1st in WA

2009	Senior Mathematics Contest (Distinction) - 3rd in WA
2008, 2009	Australian Intermediate Mathematics Olympiad (High Distinction) - 1st in WA

Since receiving the Phill Schultz Prize, Angel has produced an exceptional Tournament of the Towns result; more on that, below. Since 2007, Special WAMOC Awards have been awarded to support WA students who have been invited to the School of Excellence. The three WA students who received the 2010 Special WAMOC Awards are as listed below, along with their Australian Intermediate Mathematics Olympiad (AIMO) or Senior Mathematics Competition (SMC) scores that earned them their School of Excellence invitation:

Alexander Chua	(Year 9)	28 (Prize) SMC, 29 (High Distinction) AIMO
Kathleen Dyer	(Year 10)	19 (Distinction) SMC, 25 (High Distinction) AIMO
Edward Yoo	(Year 9)	25 (High Distinction) AIMO

It should be noted that Angel Yu was also invited to the School of Excellence as a Senior, and is a hot prospect for being a member of Australia's 2011 International Mathematics Olympiad team. Alexander Chua was also invited as a Senior and so also has a chance of making the team, and will still be around for 2012 and 2013!

TT, Northern Autumn round, for 2010, was held on Saturday, 27 November (O Level paper) and Saturday, 4 December (A level paper). The Tournament of the Towns is an "invitation-only" maths competition; a first invitation for students was made on the basis of a strong AIMO result or a significant WAJO achievement. The O paper is a 4-hour paper with five questions, and the A paper is a 5-hour paper with seven questions. A student's score on a paper is the highest total for their attempts at three of the questions. A student's overall score for the TT round is the higher score of the two papers. The three highest ranked juniors and the two highest ranked seniors have had their papers forwarded to Moscow for a more rigorous marking; and hopefully they will receive a Diploma from the Russian Academy of Sciences, to go with their certificate from the Australian Mathematics Trust. A summary of these results in order of rank is below. The reason for the Participations above the Credits in the table, are due to the requirement of needing to get out a complete question before a Credit or higher can be awarded.

Junior Student	*Year*	*School*	*Result*	*WA Rank*
Alexander Chua	9	Christ Church GS	Distinction	=1
Edward Yoo	9	All Saints' College	Distinction	=1
Andrew Yang	10	Rossmoyne SHS	Distinction	3
Bryce Lim	9	Christ Church GS	Participation	4
Daryl Chung	9	Perth Modern School	Participation	5
Andy Truong	10	Aquinas College	Credit	=6
Michael Warton	10	Hale School	Credit	=6
Diffy Zhou	9	Perth Modern School	Participation	8
Ciaran Murray	9	Trinity College	Participation	=9
Joseph Thompson	9	Perth Modern School	Participation	=9

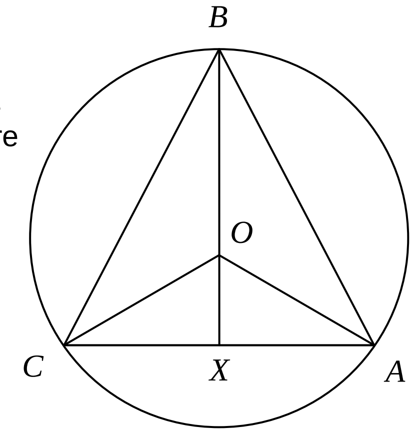
Senior Student	*Year*	*School*	*Result*	*WA Rank*
Angel Yu	11	Perth Modern School	High Dist'n	1
Calum Braham	11	Trinity College	Distinction	2
Bojana Surla	11	Penrhos College	Credit	3
Li Kho	11	Willetton SHS	Credit	4
Aaron Wong	11	Aquinas College	Credit	5
Jonathan Chung-Wah-Cheong	11	Trinity College	Participation	6
Benjamin McAllister	12	Christ Church GS	Participation	7

The next Olympiad event is the Australian Mathematics Olympiad (AMO) which will be held on Tuesday 15 and Wednesday 16 February. The four School of Excellence invitees are automatically invited. There will be a report on the results of the 2011 AMO in the next column. Finally, let us close with a problem from each of the two Olympiads featured in this column.

Question 10 (WAJO):
The area of the circumcircle of an equilateral triangle is 12π.
What is the perimeter of the triangle?

Solution. Let the triangle be ABC, with circumcentre O and circumradius R i.e. $R = OA = OB$, and denote the foot of the altitude dropped from B by X. Then

$$12\pi = \pi R^2$$
$$R = \sqrt{12} = 2\sqrt{3}$$



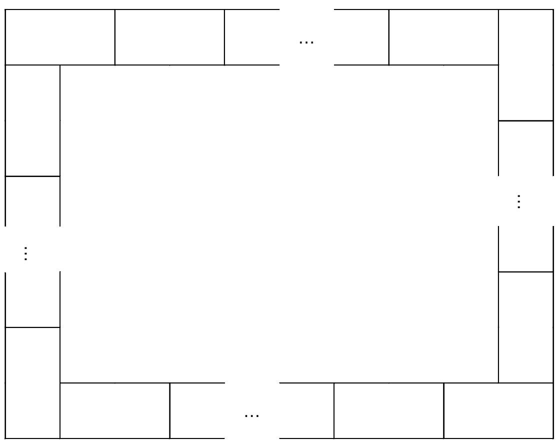
The medians of an equilateral triangle are also its altitudes and concur at O (which is also the centroid and orthocentre of the triangle). We have

$$2\sqrt{3} = R = OB = \frac{2}{3}XB$$
$$\therefore XB = 3\sqrt{3}$$

Now $AX : XB : BA = 1 : \sqrt{3} : 2$. So $AX = 3$, and hence
$AB + BC + CA = 6 \times 3 = 18$.
So the required perimeter is 18.

Question 1 (TT, Northern Autumn 2010, Junior O Level)**:**
In a multiplication table, the entry in the ith row and the jth column is the product ij. From an $m \times n$ subtable with both m and n odd, the interior $(m-2)\times(n-2)$ rectangle is removed, leaving behind a frame of width 1. The squares of the frame are painted alternately black and white. Prove that the sum of the numbers in the black squares is equal to the sum of the numbers in the white squares.

Solution. Without loss of generality, assume the lefthand top corner is a black square. Then, since the side dimensions are odd, all the corner squares of the frame are black. If we negate the values of each of the white squares, then we must show that the sum of the total value of the black squares and the total value of the white squares cancels to zero. Now imagine adjacent pairs of squares of the $m \times n$ frame are covered by dominoes as shown in the diagram, and consider the value covered by each domino.



Let the top and bottom rows of the frame be t and b, respectively, and the leftmost and rightmost columns of the frame be l and r, respectively, so that $t = b + m - 1$ and $r = l + n - 1$. Suppose a left-right domino in row t, covers squares with coordinates (t, j) and $(t, j+1)$, then the value covered by each such domino is

$$tj - t(j+1) = -t.$$

Similarly, the up-down oriented dominoes of column r, each "have value" $-r$, the bottom row left-right dominoes each have a value of b, the up-down oriented dominoes of column l each have value l.

Finally, there are $(n-1)/2$ left-right dominoes in each of rows t and b, and $(m-1)/2$ up-down dominoes in each of columns l and r. So the total value covered by the dominoes is:

$$\begin{aligned}\frac{n-1}{2}\times(-t)+\frac{n-1}{2}\times b+\frac{m-1}{2}\times(-r)+\frac{m-1}{2}\times l &= -\frac{n-1}{2}\times(t-b)+\frac{m-1}{2}\times(l-r)\\ &= -\frac{n-1}{2}\times(m-1)+\frac{m-1}{2}\times(n-1)\\ &= 0.\end{aligned}$$

Thus the total value of the black squares of the frame is equal to the total (unsigned) value of the white squares of the frame, as required.

Note. More elegantly, we may find the "value" of the entire $m \times n$ subtable, which one can show is the "value" of row t, which turns out to be $(l+r)/2$, times the "value" of column l, which similarly is $(t+b)/2$. Hence the "value" of the entire $m \times n$ subtable is

$$\frac{l+r}{2}\times\frac{t+b}{2}$$

which is the value of the middle square of the $m \times n$ subtable. Observe, now that this entire calculation equally applies to the interior $(m-2)\times(n-2)$ subtable. Since these two tables have the same "value", the frame itself must have "value" 0, as before.

***Errata*.** Unfortunately, in the two previous columns some gremlins crept into the final copy:

July 2010 (Vol. 20, No. 3) p16. In the Solution of Question 1, the subscripting for p_k, a_k, q_k, was lost (so they appeared as products of p and k, etc.). The intended displayed equations, a further two paragraphs on, were:

$$p_k = a - a_k$$

$$a_k = q - q_k\,.$$

The intended relationship between p_k and p, was that

$$\sum_{k=1}^{2010} p_k = p$$

and similarly for the other variables, though the article avoided sigma notation.

October 2010 (Vol. 20, No.4) p16. Towards the end of the Question 4 (SMC) solution, two ellipses became "L"s and two i subscripts became 1s. The intended displayed congruences were:

$$x_1 + \cdots + x_i \equiv x_1 + \cdots + x_i + x_{i+1} + \cdots + x_j \text{ (mod 5)}$$

$$0 \equiv x_{i+1} + \cdots + x_j \text{ (mod 5)}$$

Hopefully, that makes that solution a little less mysterious.