

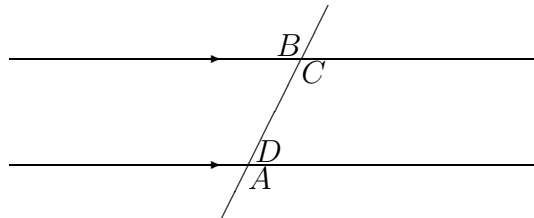
TOURNAMENT OF THE TOWNS, 2003–2004

Plane Geometry Notes

Theorem 1. *If a triangle is isosceles (i.e. it has two equal sides) then the angles opposite the equal sides are equal. Also, if two angles of a triangle are equal then the two sides opposite the equal angles are equal, so that the triangle is isosceles.*

Lines and angles

In the following diagram the two horizontal lines are parallel. The line cutting the two parallel lines is called a *transversal*. Angles A and B are called *alternate angles*, A and C are *corresponding angles*, and angles A and D are *supplementary angles*. Alternate angles and corresponding angles are equal, and pairs of supplementary angles sum to 180° .



Theorem 2. *The sum of the interior angles of a triangle is 180° .*

Theorem 3. *An exterior angle of a triangle equals the sum of the two non-adjacent interior angles.*

Theorem 4. *The sum of the interior angles of an n -sided polygon is $180(n - 2)^\circ$.*

Quadrilaterals and parallelograms

Theorem 5. *The opposite sides and opposite interior angles of a parallelogram are equal.*

Theorem 5a. *If a quadrilateral has opposite sides equal then it is a parallelogram.*

Theorem 5b. *If a quadrilateral has opposite interior angles equal then it is a parallelogram.*

Theorem 6. *The diagonals of a parallelogram bisect each other.*

Theorem 7. *The diagonals of a rhombus are perpendicular.*

Special Triangle Theorems

Theorem 8. (Pythagoras' Theorem) *In a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

Theorem 9. (Sine Rule) *In a triangle ABC where a, b, c are the lengths of the sides opposite the vertices A, B, C , respectively,*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Theorem 10. (Cosine Rule) *In a triangle ABC where a, b, c are the lengths of the sides opposite the vertices A, B, C , respectively,*

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Congruence of triangles

Triangles may be determined to be congruent by any of the following rules.

- **SSS Rule** If three sides of one triangle are equal to the three sides of another, then the triangles are congruent.
- **SAS Rule** If two sides and the included angle of one triangle are equal to the two sides and the included angle of another, then the triangles are congruent.
- **ASA Rule** If two angles and the included side of one triangle are equal to the two angles and the included side of another, then the triangles are congruent.
- **RHS Rule** If the hypotenuse and one other side of a right-angled triangle are equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Note that when we say two triangles ABC and XYZ are *congruent* we mean that the correspondence of vertex A to X , B to Y and C to Z determines the congruence. We denote that two triangles ABC and XYZ are *congruent* by writing $\triangle ABC \cong \triangle XYZ$.

Similarity of triangles

Each of the congruence rules has a corresponding similarity rule, by replacing side-length equality by proportionality. Thus, triangles may be determined to be similar by any of the following rules.

- **SSS Rule** If three sides of one triangle are in the same proportion as the three sides of another, then the triangles are similar.
- **SAS Rule** If two sides of one triangle are in the same proportion as the two sides of another, and the included angles of the sides that correspond are equal then the triangles are similar.
- **AA Rule** (or **AAA Rule**) If two angles of one triangle are equal to two angles of another, then the triangles are similar. (The equality of the two remaining corresponding angles are then necessarily equal.)
- **RHS Rule** If the hypotenuse and one other side of a right-angled triangle are in the same proportion as the hypotenuse and one side of another right-angled triangle, then the triangles are similar.

As with congruence, when we say two triangles ABC and XYZ are *similar* we mean that the correspondence of vertex A to X , B to Y and C to Z determines the similarity. We denote that two triangles ABC and XYZ are *similar* by writing $\triangle ABC \sim \triangle XYZ$.

Theorem 11. *If a line joins the midpoints of two sides of a triangle then that line is parallel to the third side and its length is equal to one half of the length of the third side.*

Theorem 12. *A line parallel to one side of a triangle divides the other two sides in the same proportion.*

Theorem 13. *The bisector of one side of a triangle divided the opposite side in the same ratio as the other two sides.*

Areas and perimeters

Theorem 14. The area of a parallelogram is equal to bh where b is the length of its base and h is its height (the perpendicular distance from the base to the parallel side opposite).

Theorem 15. The area of a triangle is equal to $\frac{1}{2}bh$ where b is the length of its base and h is its height (the perpendicular distance from the base to the vertex opposite).

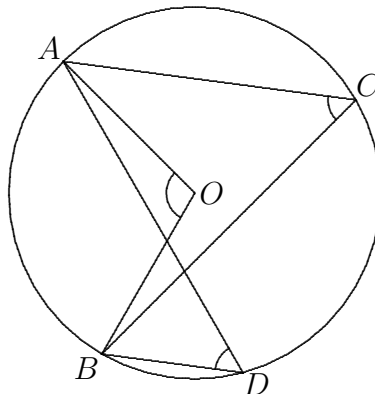
Theorem 16. (Heron's Theorem) If the lengths of the sides of a triangle are a , b and c , so that the semiperimeter $s = (a + b + c)/2$ then the area of the triangle is

$$\sqrt{s(s-a)(s-b)(s-c)}$$

Theorem 17. The area of a circle of radius r is πr^2 and its circumference is $2\pi r$.

Circles

Theorem 18. If AB is an arc of a circle then angles subtended at the circumference opposite AB are equal and are equal to half the angle subtended at the centre, i.e. in the diagram $\angle ACB = \angle ADB = \frac{1}{2}\angle AOB$.



Theorem 19. If AB is an arc of a circle and C is any point on the circumference of the circle then $\angle ACB$ is a right angle.

Theorem 20. If A and B are points on the circumference of a circle with centre O and C is an exterior point of the circle such that BC is a tangent to the circle then $\angle ABC = \frac{1}{2}\angle AOB$.

Theorem 21. A line from the centre of a circle perpendicular to a chord bisects the chord and its arc.

Theorem 22. If A is a point on the circumference of a circle then a tangent to the circle at A is perpendicular to a radius of the circle to A .

Theorem 23. The two tangents drawn to a circle drawn from an exterior point of the circle have the same length.

Theorem 24. *If two circles touch at a single point then this point and the centres of the circles are collinear.*

Theorem 25. *If two circles intersect at two points then the line through their centres is the perpendicular bisector of their common chord.*

Theorem 26. *Opposite angles of a cyclic quadrilateral sum to 180° and if a pair of opposite angles of a quadrilateral sum to 180° then it is cyclic.*

Theorem 27. *The centre of the circumcircle of a triangle is the intersection of the perpendicular bisectors of the sides of the triangle.*