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Glossary

Absolute value The "size" of a number with its + or − sign removed. The *absolute value* of −3.2 is 3.2, the absolute value of +4.6 is 4.6. We write this: $|-3.2| = 3.2$ and $|4.6| = 4.6$. (A + sign in front of a number is superfluous.)

Altitude The line through a vertex of a triangle that is perpendicular to the opposite side. A triangle has three *altitudes;* they are concurrent, meeting at the triangle's *orthocentre*.

Arc Any portion of the circumference of a circle.

Binomial coefficient A number that appears in Pascal's triangle (the first 5 rows of which appear below)

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

The number that appears in the nth row and rth diagonal, numbering from 0 (so that the 1 at the apex of the triangle is in the zeroth row and zeroth diagonal), is the integer coefficient of $x^r y^{n-r}$ when the binomial $(x + y)^n$ is expanded. We write this *binomial coefficient* in either of the following ways

$$\binom{n}{r} \quad \text{or} \quad {}^nC_r$$

where the C in the second notation stands for *combination* or *choose*. It is equal to the number of ways we can choose a set of r objects from n objects (so we often read $\binom{n}{r}$ as *n choose r*), e.g. we can choose 2 objects (without repetition and neglecting the order in which they were chosen) from 4 objects in 6 ways:

If the objects A, B, C, D we have AB, AC, AD, BC, BD, CD.

The most important properties of $\binom{n}{r}$ are

- $\displaystyle\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$
- $\displaystyle\binom{n}{r} = \binom{n}{n-r}$
- $\displaystyle\binom{n}{0} = 1 = \binom{n}{n}$
- $\displaystyle\binom{n}{1} = n = \binom{n}{n-1}$
- $\displaystyle\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1}$

Centroid The point at which the three medians of a triangle concur. The *centroid* trisects each of the medians, i.e. splits each median in the ratio $2 : 1$.

Chord A line segment whose endpoints lie on the circumference of a circle.

Circumcentre, circumcircle The three perpendicular bisectors of the sides of a triangle concur at the *circumcentre* of the triangle, which is the centre of the *circumcircle*, the circle that passes through the three vertices of the triangle.

Collinear This means *lying on the same straight line*. Several points are *collinear* if you can draw a straight line through all of them.

Combination A choice of a fixed number of objects from a larger set of objects, without regard to ordering. For the objects A, B, C, D each of AB, AC, AD, BC, BD, CD is a *combination of the four objects* A, B, C, D *taken* 2 *at a time*. A combination differs from a permutation, in that order is not important, i.e. AB and BA represent the same combination, but are different permutations.

Composite An integer that has positive divisors other than the (absolute value of) itself and 1. In particular, natural numbers greater than 1 that are not prime are *composite*, e.g. $4, 6, 8, 9, \ldots$.

Concurrent This means *going through the same point*. Several lines are *concurrent* if they all intersect in the same point.

Congruent Two polygons are *congruent* if they have the same size and shape (i.e. if one were to shift and/or reflect one polygon the vertices of the two polygons could be made to line up exactly); in particular corresponding sides are of the same length.

Convex A set S of points on a line, plane or in space is *convex* if for any points A, B in S, all points on the line segment AB are in S. We say a polygon is *convex* if any line segment between points on the boundary of the polygon only intersects the interior of the polygon, i.e. all its interior angles are less than $180°$, e.g. any regular polygon is convex.

Coprime Another term for *relatively prime*. Two numbers are *coprime* if their *greatest common divisor* is 1.

Cyclic A quadrilateral is *cyclic* if a circle may be drawn that passes through each of its four vertices.

Diameter A chord of a circle that passes through the circle's centre.

Divisor Same as a *factor*. An integer that divides (evenly) into another, e.g. each of 1, 3, 5 and 15 and their negatives is a *divisor* (or *factor*) of 15.

Edge A side of a geometrical figure, or more generally, a line segment that joins two vertices.

Equilateral A triangle is *equilateral* if all its sides are of equal length. An equilateral triangle necessarily has all its angle equal to $60°$.

Factor Same as a *divisor*.

Factorial n *factorial* (written $n!$) is the product of all the natural numbers from 1 up to n, e.g. 6 *factorial* (written 6!) means $1\times2\times3\times4\times5\times6$. Thus $3! = 1\times2\times3 = 6$, which is th number of ways we can line up 3 objects, i.e. the number of *permutations* of 3 objects, e.g. if the objects are A, B, C we have the permutations $ABC, ACB, BAC, BCA, CAB, CBA$.

Greatest common divisor, gcd Same as *highest common factor (hcf)*. The largest natural number that divides each of a given set of two or more integers, e.g. $\gcd(-12, 15) = 3$ and $\gcd(16, 24, 60) = 4$.

Highest common factor, hcf Same as *greatest common divisor (gcd)*.

Incentre, incircle The three internal bisectors of the angles of a triangle concur at the *incentre* of the triangle, which is the centre of the *incircle*, the circle that touches each side of the triangle, i.e. each side of the triangle is a tangent to the incircle.

Integer A whole number, i.e. any of $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$. In particular, *integers* can be positive, negative or zero. The set of all integers is denoted by $\mathbb{Z}$ (for the German *zahlen* which means *numbers*).

Isosceles A triangle is *isosceles* if two of its sides are of equal length, in which case, the two angles not included by the sides of equal length are equal.

Least common multiple, lcm The smallest natural number that is an integer multiple of each of a given set of integers. In particular, the lowest common denominator (lcd) of a set of fractions is the lcm of the denominators of those fractions.

Line Always means a *straight line* that is infinite in both directions.

Line segment A piece of a line of a definite length with two ends.

Locus The line, curve or region traced out by a point satisfying certain conditions, e.g. if a point moves with fixed distance from a fixed point then its *locus* is a circle.

Median A line joining the vertex of a triangle to the midpoint of the opposite side. A triangle has three *medians*; they concur at the *centroid* of the triangle.

Natural number A positive integer, i.e. one of $1, 2, 3, \dots$. The set of all *natural numbers* is denoted by $\mathbb{N}$.
Caution. Some people also consider 0 to be a natural number.

Negative Less than zero, e.g. $-1, -1.2, -3$ are *negative*, but 0 is not negative.

Non-negative Positive or zero, e.g. $0, 1.1, 2$ are *non-negative*. In particular, 0 is *non-negative* but not positive.

Non-positive Negative or zero, e.g. $0, -1.1, -2$ are *non-positive*. In particular, 0 is *non-positive* but not negative.

Orthogonal Same as *perpendicular*.

Orthocentre The common intersection point of the three altitudes of a triangle.

Parallelogram A quadrilateral that has two pairs of parallel sides.

Parallelepiped A solid object with six faces, each face being a parallelogram. In a rectangular *parallelepiped* (also known as a *rectangular prism*) each face is a rectangle.

Period, periodic A *periodic* function is one that repeats itself regularly, in the sense that there exists a number p such that

$$f(x + p) = f(x)$$

for all values of x. Such a number p is called a *period* for the function f. In general if p is a period for a function f, then every positive integer multiple of p is also a period for f. The least positive p that is a period for f, is often referred to as *the* period for f.

Permutation A way of putting a set of objects into order, e.g. the letters A, B, C can be put in the orders $ABC, ACB, BAC, BCA, CAB, CBA$. So there are 6 permutations of A, B, C. Generally, there are $n! = n \times (n-1) \times \cdots \times 1$ permutations of n objects.

Perpendicular At right angles.

Plane figure A geometrical figure consisting of vertices and edges that can be drawn in the plane; a 2-dimensional object.

Polygon A plane figure whose edges are connected end to end in a loop. A *polygon* with n sides is sometimes called an *n-gon.* (Technically, a *gon* is an angle, but an *n-gon* has just as many sides as it has angles, so could just as easily have been called an n-lateral.) *Trigon* and *trilateral* are uncommon synonyms for *triangle.* 4-gons are generally referred to as *quadrilaterals* and sometimes as *quadrangles.* And we have *pentagon* (5-gon), *hexagon* (6-gon), *heptagon* (7-gon), *octagon* (8-gon), *nonagon* (9-gon), *decagon* (10-gon), *dodecagon* (12-gon), etc.

Polyhedron (plural: **polyhedra**) A solid bounded by plane faces. A *polyhedron* is regular if all its faces are congruent regular polygons and each vertex is incident with the same number of edges. There are just five regular polyhedra, the so-called *Platonic solids:* regular tetrahedron, cube, regular octahedron (which has 8 faces that are equilateral triangles), regular dodecahedron (which has 12 faces that are regular pentagons), and regular icosahedron (which has 20 faces that are equilateral triangles).

Polynomial An expression like $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where the a_i are numbers. The highest power of x, in this case n so long as a_n is nonzero, is called the *degree* of the *polynomial.* The numbers a_i are called *coefficients.*

Positive Greater than zero, e.g. $1, 1.2, 3$ are *positive*, but 0 is not positive.

Pyramid A polyhedron with four isosceles triangular sides and a square base. An *oblique pyramid* has four triangular sides but they need not all be isosceles. A *triangular pyramid* (same as a *tetrahedron*) has a triangular base, i.e. all faces are triangular none of which need be isosceles.

Prime A *prime* or *prime number* is a natural number larger than 1 that is divisible only by 1 and itself. In particular, 1 is *not* a prime.

Radius (plural: **radii**) A line segment from the centre to the circumference of a circle.

Rational number A number that can be written in the form p/q where both p and q are integers. The set of all *rational numbers* is denoted by $\mathbb{Q}$ (think of q for *quotient*). Integers and fractions are rational numbers but numbers such as π and $\sqrt{2}$ are not. Numbers that are not rational are *irrational.*

Real number Any rational number or irrational number. The set of all *real numbers* is denoted by $\mathbb{R}$.

Reciprocal The *reciprocal* of a number is 1 divided by that number, e.g. the reciprocal of 5 is $\frac{1}{5}$, and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Regular A polygon is *regular* if all its sides are equal and all its angles are equal.

Relatively prime Same as *coprime.*

Rhombus A parallelogram whose sides are all of equal length.

Secant A line that intersects a circle in two distinct points.

Sector The area bounded by an arc of a circle and the two radii joining the arc.

Sequence A (finite or infinite) list of numbers that may or may not have a pattern, e.g. $1, 3, 5, 7$ or $1.2, 2.3, 4.9, \ldots$. Many famous sequences are both infinite and have a pattern, e.g. the *Fibonacci sequence,* $1, 1, 2, 3, 5, 8, \ldots$ satisfies the *recurrence relation* $F_{n+2} = F_n + F_{n+1}$ where F_n is the nth member of the sequence.

Similar Two polygons are *similar* if angles at corresponding vertices are equal (if the two polygons are $ABC \ldots$ and $XYZ \ldots$ then A corresponds to X, B corresponds to Y, etc.), in which case corresponding sides are in the same proportion.

Simple A *simple* plane figure is one that does not cross itself.

Tangent A line in the same plane as a circle that intersects (i.e. touches) the circle at exactly one point.

Tetrahedron A polyhedron with 4 faces, all of which are triangles. The same as a *triangular pyramid.*

Trapezium, trapezoid A quadrilateral that has one pair of opposite sides parallel.

Vertex A "corner" of a geometrical figure, i.e. a point at which edges meet.

Without loss of generality, w.l.o.g. It means we can make a simplification of a problem without our proof losing its application to the whole problem, e.g. suppose a problem begins "Suppose we have two numbers ...", the our proof might begin:

> Let the 2 numbers be x and y and suppose, *without loss of generality*, that $x \geq y$.

One of the numbers must be as least as big as the other; so we may as well say that x is the bigger one.