Western Australian
Junior Mathematics Olympiad 2000

Individual Questions **100 minutes**

General instructions: *No working need be given for Questions 1 to 9. Calculators are not permitted. Write your answers on the answer sheet provided.*

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(1) How many non-overlapping equilateral triangles with 1 centimetre sides can we fit inside an equilateral triangle with 10 centimetre sides? [1 mark]

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(2) Four accused people face trial. It is known that:
(a) If A is guilty, then B is guilty,
(b) If B is guilty, then C is guilty or A is not guilty,
(c) If D is not guilty, then A is guilty and C is not guilty,
(d) If D is guilty, then A is guilty.
How many of the accused must be guilty? [1 mark]

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(3) Find the number of integers between 100 and 500 such that the sum of their digits is 10. [1 mark]

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(4) In the expression $S = a - b + c - d$ the symbols a, b, c, d are replaced by 1, 2, 3, 4 in any order with no repetitions allowed. There are 24 possible replacements.
In how many of these will S be greater than 0? [1 mark]

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(5) Let ABC be an acute angled triangle. Let M be a point on BC such that AM is perpendicular to BC. Let N be a point on AB such that CN is perpendicular to AB. If H is the intersection point of AM and CN and it is given that $HM = HN$ and $BC = 20$, find AB. [2 marks]

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(6) The visibility at sea, on a certain day, is 5 kilometres. Ships A and B (which start a long way apart) are travelling in opposite directions on courses which are parallel and 3 kilometres apart. The two ships are in sight of one another for 24 minutes.
If ship A is travelling at 8 kilometres per hour, how fast is ship B travelling? [2 marks]

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(7) How many integers between 100 and 1000 are there that are not exactly divisible by 2, 3 or 5? [2 marks]

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(8) The number 6 is divisible by 1, 2, 3 and 6; so 6 has 4 divisors.
How many divisors has $6718464 = 2^{10} \times 3^8$? [2 marks]

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(9) Let M be the point on the extension of the side BC of a parallelogram $ABCD$ such that B is between M and C and $MB = BC$.
If P and N are the intersection points of MD with AC and AB, respectively, and $PN = 3$, find DP. [2 marks]

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(10) Ten students have altogether 35 coins. It turns out that at least one of them has exactly one coin, at least one has exactly two coins, and at least one has exactly three coins. Explain why you can be sure that at least one student has 5 or more coins. [4 marks]

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Team Questions **45 minutes**

(1) A cube, consisting of 125 cubes each with side 1 centimetre, is drilled through in three places. The holes are rectangular-shaped with cross-section 1 centimetre by 3 centimetres (see picture) and go all the way through the large cube. How many small cubes remain after the drilling?

(2) Instead of a $5 \times 5 \times 5$ cube, say you have a $7 \times 7 \times 7$ cube, with three slots having cross-section 1 centimetre by 5 centimetre drilled through the middle. How many small cubes remain?

(3) Now suppose you have an $n \times n \times n$ cube, with n an odd number. Three slots, each with cross-section 1 centimetre by $n - 2$ centimetres are drilled through the middle. Find a formula for the number of small cubes that remain.

(4) Briefly explain how you obtained your answer to question 3.

Write your answers on the accompanying sheet.