Western Australian
Junior Mathematics Olympiad 2017

Individual Questions **100 minutes**

General instructions: *Except possibly for Question 12, each answer in this part is a positive integer less than 1000. No working is needed for Questions 1 to 11. Calculators are* ***not*** *permitted. In general, diagrams are provided to clarify wording only, and are not to scale.*
Write your answers for Questions 1–11, and solution for Question 12 on the front and back, respectively, of the Answer Sheet provided.

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1. Let ABC be a right-angled triangle with right angle at A. Let D be on the line BC, with B between C and D.
If the angle $\angle ACB$ is 6 degrees more than a quarter of the angle $\angle ABD$, how many degrees is $\angle ABC$? [1 mark]

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2. When Geppetto adds 6 times his age 6 years from now to 7 times his age 7 years from now, he gets 14 times his current age.
How old is Geppetto now? [1 mark]

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3. The squares of two consecutive integers differ by 2017.
Of the two consecutive integers, one is odd and one is even; what is half of the even one? [1 mark]

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4. How many positive integers y are there such that the least common multiple of 280 and y is 1400? [2 marks]

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5. A two-digit number ab is called *good* if $a^2 + b^2 = 65$.
What is the sum of all *good* numbers? [2 marks]

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6. The average of the ages of the mother, the father and their three children is 21, whereas the children alone average 11.
How many years is the age of the father, given that the father is 4 years older than the mother? [2 marks]

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7. If $x^2 + xy + y^2 = 92$ and $x + y = 11$, what is xy? [2 marks]

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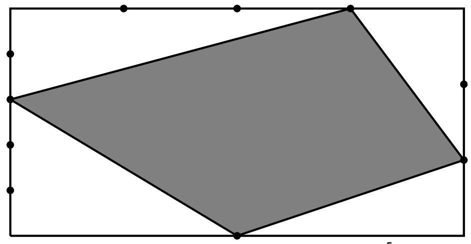
8. The four sides of a rectangle of area 630 are divided into halves, thirds, quarters and fifths respectively, as in the diagram.

What is the area of the shaded quadrilateral? Enter 999 if you think it depends on the lengths of the sides of the rectangle.



[2 marks]

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9. Ayesha and Brian live 5 km apart and plan to meet at 1:00 pm. They decide to leave their respective homes at the same time and cycle towards each other till they meet. Ayesha cycles at 8 km/h and Brian at 7 km/h.

How many minutes after 12 noon should they leave home? [2 marks]

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10. A block of wood in the form of a $5\,\text{cm} \times 8\,\text{cm} \times 13\,\text{cm}$ rectangular prism has all six of its faces painted pink.

If the wooden block is cut into $1\,\text{cm}^3$ cubes, how many of these would have some pink paint on them? [3 marks]

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11. Janine has a box containing 49 red balls and 49 green balls and a jar containing one red ball and one green ball.

She performs the following multi-step operation:

(i) First she closes her eyes, takes one ball from the jar, looks at it, notes its colour, and puts it in the box.

(ii) If the ball moved to the box was *red*, she closes her eyes, takes *two balls* from the box and puts them in the jar;
otherwise, if the ball moved to the box was *green*, she takes *two red balls* from the box and puts them in the jar.

Suppose Janine repeats this operation 36 times and finds there are 36 red balls in the box, how many green balls will there be in the box? [3 marks]

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12. For full marks explain how you found your solution.

Two sides of an obtuse triangle have lengths 9 and 40.

The length of the third side is an integer.

How many such triangles are possible?

Note. An *obtuse triangle* is a triangle with one obtuse angle.

An angle θ is *obtuse* if $90^\circ < \theta < 180^\circ$. [4 marks]

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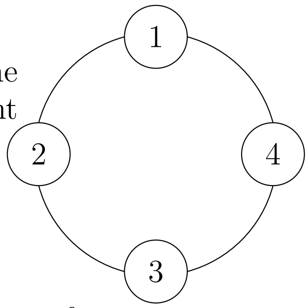
Team Question **45 minutes**

General instructions: *Calculators are (still)* ***not*** *permitted.*

Prime Circles for Nora

Nora writes the numbers 1, 2, 3 and 4 in a circle as in the diagram opposite, and notices that each pair of adjacent numbers has a prime sum:

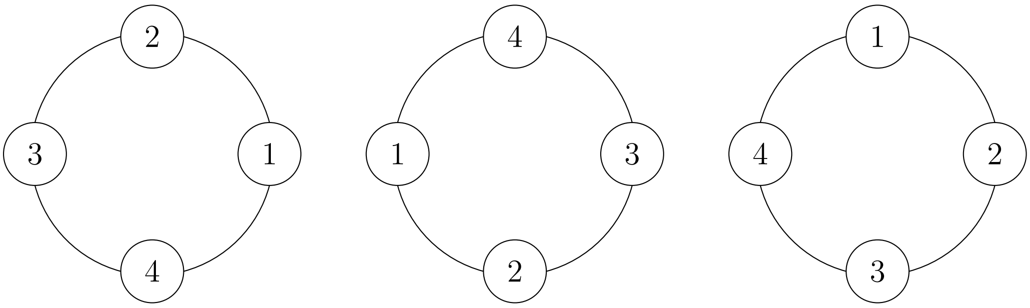


$$1 + 2 = 3,\ 2 + 3 = 5,\ 3 + 4 = 7,\ 4 + 1 = 5.$$

We call a *prime circle for n* an arrangement of the integers from 1 to n around a circle such that the sum of any two adjacent numbers is a prime.

Two prime circles for n are considered the *same* (or *equivalent*) if one can be obtained from the other by a rotation or a reflection. Otherwise we say the circles are *different*.

For instance, the three circles below are equivalent to the one above, via a clockwise rotation, an anticlockwise rotation, and a reflection, respectively.



Clearly, using rotations we can decide that the number 1 is in a fixed location, for instance at "12 o'clock" as on the answer sheet. Moreover, using the reflection in the line through 12 and 6 o'clock, two circles that have the numbers in the same order starting with 1, but one clockwise and one anti-clockwise, are equivalent.

Finally, you might find convenient to write a prime circle for n as a sequence of n numbers, starting at 12 o'clock and going clockwise. For example, the first and the last prime circles above can be written as $(1, 4, 3, 2)$ and $(1, 2, 3, 4)$ respectively.

A. How many different prime circles for 4 are there?

B. Find a prime circle for 6.

C. How many different prime circles for 6 are there?

D. If n is odd, explain why there is no prime circle for n.

E. Find a prime circle for 8.

F. How many different prime circles for 8 are there?

G. Find a prime circle for 10.

H. Show that if you swap 3 and 9 from a prime circle for 10 you get a different prime circle for 10.

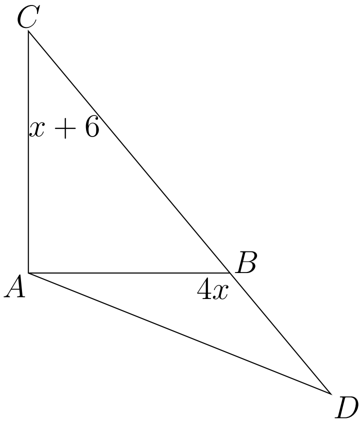
I. Are there other numbers that can be swapped in a prime circle for 10?

J. Show that the number of different prime circles for 10 is divisible by 4.

K. Show that the number of different prime circles for 12 is divisible by 16.

Individual Questions Solutions

1. Answer: 52.



Let $\angle ABD = 4x$.
Then $\angle ACB = x + 6$ and
we want $\angle ABC = 180° - 4x$.
Since for a triangle, an exterior angle is the sum of the interior opposite angles,

$$\begin{aligned} 4x &= x + 6 + 90 \\ 3x &= 96 \\ x &= 32 \\ \therefore \angle ABC = 180° - 4x &= 180° - 128° \\ &= 52° \end{aligned}$$

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2. Answer: 85. Let Geppetto's age now be x. Then

$$\begin{aligned} 6(x+6) + 7(x+7) &= 14x \\ 36 + 49 &= x \\ \therefore x &= 85. \end{aligned}$$

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3. Answer: 504. Let $n, n+1$ be the consecutive integers. Then

$$\begin{aligned} 2017 &= (n+1)^2 - n^2 \\ &= 2n + 1 \\ &= n + (n+1) \\ \therefore n &= 2016/2 \\ &= 1008 \end{aligned}$$

i.e. the even one of the two consecutive integers is $n = 1008$ and half of it is 504.

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4. Answer: 8.
$1400 = 8 \cdot 25 \cdot 7 = 2^3 \cdot 5^2 \cdot 7$ and $280 = 2^3 \cdot 5 \cdot 7$.
Hence $y = 2^a \cdot 5^b \cdot 7^c$, where we require

$$\begin{aligned} \max\{3, a\} = 3 &\implies a = 0, 1, 2 \text{ or } 3 && \text{(4 possibilities)} \\ \max\{1, b\} = 2 &\implies b = 2 && \text{(1 possibility)} \\ \max\{1, c\} = 1 &\implies c = 0 \text{ or } 1 && \text{(2 possibilities)} \end{aligned}$$

Thus the number of choices for the triple (a, b, c) (and hence of y) is $4 \times 1 \times 2 = 8$.

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5. Answer: 220. Since $9^2 = 81$, the digits $a, b \in \{1, 2, \ldots, 8\}$.
The larger of the 2 digits is at least 6, since $5^2 < \frac{1}{2} \cdot 65$.
Now $65 - 6^2 = 29$, is not a square.
This leaves checking whether 7 and 8 could be one of the digits. Indeed,

$$7^2 + 4^2 = 65 = 8^2 + 1^2.$$

So there are 4 good numbers: 18, 47, 74, and 81, and their sum is

$$18 + 47 + 74 + 81 = 220.$$

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6. Answer: 38. Let m, d, c_1, c_2, c_3 be the ages of the mother, father and three children, respectively. Then

$$\begin{aligned}\tfrac{1}{5}(d+m+c_1+c_2+c_3) &= 21\\ \text{and } \tfrac{1}{3}(c_1+c_2+c_3) &= 11\\ \therefore \tfrac{1}{5}(d+m+3\cdot 11) &= 21\\ d+m+33 &= 105\\ d+m &= 72\\ d+d-4 &= 72, \quad \text{since } d = m+4\\ 2d &= 76\\ d &= 38.\end{aligned}$$

So the father is 38.

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7. Answer: 29.

$$\begin{aligned}xy &= (x^2+2xy+y^2)-(x^2+xy+y^2)\\ &= (x+y)^2-(x^2+xy+y^2)\\ &= 11^2-92\\ &= 121-92 = 29.\end{aligned}$$

Remark. The solutions for x, y are irrational, being the solutions of the quadratic equation

$$u^2-11u+29=0,$$

namely $\{x,y\} = \{\tfrac{1}{2}(11+\sqrt{5}), \tfrac{1}{2}(11-\sqrt{5})\}$.

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8. Answer: 336. Let the length and breadth of the rectangle be ℓ and b, respectively. Then the shaded area is the area of the rectangle minus the areas of the four unshaded triangles, i.e.

$$\begin{aligned}\text{Shaded area} &= 630 - \tfrac{1}{2}\cdot\tfrac{3}{4}\ell\cdot\tfrac{2}{5}b - \tfrac{1}{2}\cdot\tfrac{1}{2}\ell\cdot\tfrac{3}{5}b - \tfrac{1}{2}\cdot\tfrac{1}{2}\ell\cdot\tfrac{1}{3}b - \tfrac{1}{2}\cdot\tfrac{1}{4}\ell\cdot\tfrac{2}{3}b\\ &= 630\big(1-\tfrac{1}{2}(\tfrac{3}{4}\cdot\tfrac{2}{5}+\tfrac{1}{2}\cdot\tfrac{3}{5}+\tfrac{1}{2}\cdot\tfrac{1}{3}+\tfrac{1}{4}\cdot\tfrac{2}{3})\big), \text{ since } \ell b = 630\\ &= 630\big(1-\tfrac{1}{2}(\tfrac{3}{10}+\tfrac{3}{10}+\tfrac{1}{6}+\tfrac{1}{6})\big)\\ &= 630\big(1-\tfrac{1}{2}(\tfrac{3}{5}+\tfrac{1}{3})\big)\\ &= 630(1-\tfrac{1}{2}\cdot\tfrac{14}{15})\\ &= 630\cdot\tfrac{8}{15}\\ &= 21\cdot 16\\ &= 336.\end{aligned}$$

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9. Answer: 40. The total of their speeds is 15 km/h.
So they will reduce the distance 5 km between them to zero, in $\tfrac{1}{3}$h = 20 minutes.
Therefore they should leave home at 12:40 pm, i.e. 40 minutes after 12 noon.

Alternatively, suppose Ayesha cycles d km, so that Brian cycles $(5-d)$ km. Then the time t in hours they both travel is

$$\begin{aligned} t = d/8 &= (5-d)/7 \\ 7d &= 8(5-d) \\ &= 40 - 8d \\ 15d &= 40 \\ d &= \tfrac{8}{3} \\ \therefore t = d/8 &= \tfrac{1}{3}\,\text{h} \\ &= 20\,\text{min} \end{aligned}$$

and so they each leave 20 minutes before 1:00 pm and hence 40 minutes after 12 noon.

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10. Answer: 322. The number of partially pink $1\,\text{cm}^3$ cubes is the total number of cubes minus the number of cubes in the inner prism with no paint on them, i.e.

$$\begin{aligned} &\text{No. of partially pink faces} \\ &\quad = 5 \times 8 \times 13 - (5-2) \times (8-2) \times (13-2) \\ &\quad = 520 - 3 \times 6 \times 11 \\ &\quad = 520 - 198 \\ &\quad = 322 \end{aligned}$$

Alternatively, the number of partially pink $1\,\text{cm}^3$ cubes is the number in the top and bottom faces, plus the number in the front and back faces not in the top and bottom faces, plus the number in the left and right faces neither in the top or bottom, or front or back faces, i.e.

$$\begin{aligned} &\text{No. of partially pink faces} \\ &\quad = 2 \times 13 \times 8 + 2 \times 13 \times (5-2) + 2 \times (8-2) \times (5-2) \\ &\quad = 2 \times \big(13 \times (8+5-2) + (8-2) \times (5-2)\big) \\ &\quad = 2 \times (13 \times 11 + 6 \times 3) \\ &\quad = 2(143 + 18) \\ &\quad = 322. \end{aligned}$$

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11. Answer: 26. The overall effect of each operation is that one ball is added to the jar; so after 36 operations there are $2 + 36 = 38$ balls in the jar.
(Note that 36 operations can certainly be carried out, since, firstly, $36 < 98$ so that there are always sufficient balls in the box to carry out the operation. Secondly, since the number of balls in the jar is never more than 36, and one of the balls in the jar is initially red, at any one time at most 35 of the red balls initially in the box can have been removed; so that there are always more than 2 red balls in the box. Thirdly, the jar initially has 2 balls and hence never less than 1 ball.)
There are 100 balls altogether; so there are now $100 - 38 = 62$ balls in the box.
Since 36 are red the remaining $62 - 36 = 26$ are green.

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12. Answer: 14. Let the third side be c. To form a valid triangle, we require

$$9 + c > 40 \text{ and } 9 + 40 > c$$
$$\therefore 31 < c < 49$$

In order for a triangle with sides a, b and ℓ, with ℓ the longest side, to be obtuse, ℓ must be longer than the hypotenuse of a right angled triangle with legs a and b, i.e.

$$\ell^2 > a^2 + b^2$$

Thus if $c < 40$ we require

$$\begin{aligned} 40^2 &> 9^2 + c^2 \\ c^2 &< 40^2 - 9^2 \\ &= 1519 \\ c &\leq 38. \end{aligned}$$

And, if $c > 40$ we require

$$c^2 > 9^2 + 40^2.$$

Note that $9, 40, 41$ is a Pythagorean triad. To see this observe that,

$$\begin{aligned} 9^2 &= 40 + 41 \\ &= (41 + 40)(41 - 40) \\ &= 41^2 - 40^2 \\ 9^2 + 40^2 &= 41^2 \end{aligned}$$

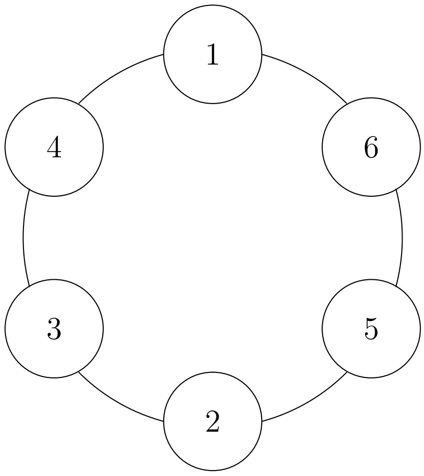
Thus, if $c > 40$, to have an obtuse triangle $c > 41$.

Therefore, we have $32 \leq c \leq 38$ or $42 \leq c \leq 48$ which is a total of 14 possibilities.

Prime Circles for Nora

A. 1 and 3 cannot be adjacent (as they add up to 4); so are opposite on the circle.
Similarly 2 and 4 cannot be adjacent (as they add up to 6); so they are opposite.
So up to rotation we get only $(1, 2, 3, 4)$ or $(1, 4, 3, 2)$, but these are the same via a reflection.
So there is only one solution.

B. Answer: $(1, 6, 5, 2, 3, 4)$.
This solution is shown in the diagram.



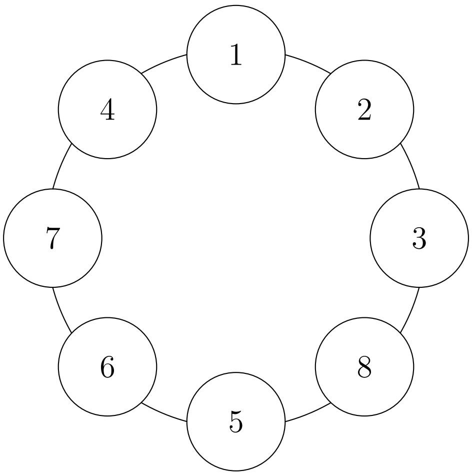
Up to equivalence the above is the only solution.

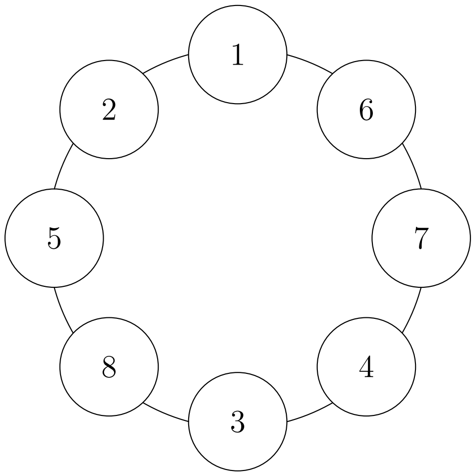
C. Two odd numbers cannot be adjacent as they would have an even sum larger than 2.
Up to rotation and reflection, we can assume a circle is $(1, x, 3, y, 5, z)$.
4 cannot be adjacent to 5, so $4 = x$.
6 cannot be adjacent to 3, so $6 = z$.
Hence $2 = y$.
There is only the one solution found in **B**.

Alternatively, two even numbers cannot be adjacent as they would sum to an even number larger than 2.
Up to rotation and reflection, we can assume a circle is $(2, x, 4, y, 6, z)$.
4 cannot be adjacent to 5, so $5 = z$.
6 cannot be adjacent to 3, so $3 = x$.
Hence $1 = y$.
There is only the one solution found in **B**.

D. For n odd, there are more odd numbers than even numbers around the circle, so two odd numbers will be adjacent.
Two positive odd numbers sum to an even number larger than 2.
So the circle cannot be a prime circle for n.

E. Answer: $(1, 2, 3, 8, 5, 6, 7, 4)$ or $(1, 6, 7, 4, 3, 8, 5, 2)$.
These two non-equivalent solutions are shown below.





Up to equivalence, these are the only correct solutions.

F. We must alternate odd and even numbers; otherwise we have adjacent numbers summing to an even number larger than 2.
Up to rotation we can have 1 at 12 o'clock.
Then up to reflection, there are only 3 possibilities for placing the other odd numbers:
(i) $(1, x, 3, y, 5, z, 7, t)$, (ii) $(1, x, 7, y, 3, z, 5, t)$, and (iii) $(1, x, 5, y, 7, z, 3, t)$.
The prime sum condition to avoiding forbidden odd/even adjacent pairs:

$$1 - 8,\ 3 - 6,\ 5 - 4,\ 7 - 2,\ \text{ and } 7 - 8.$$

Since 8 can neither be adjacent to 1 nor 7, case (iii) is impossible.
For case (i), 8 must be y, and
for case (ii), 8 has to be z (between 3 and 5 in both cases).
So we have:
(i) $(1, x, 3, 8, 5, z, 7, t)$ or (ii) $(1, x, 7, y, 3, 8, 5, t)$.
2 cannot be adjacent to 7 so we get:
(i) $(1, 2, 3, 8, 5, z, 7, t)$ or (ii) $(1, x, 7, y, 3, 8, 5, 2)$.
4 cannot be adjacent to 5 so we get:
(i) $(1, 2, 3, 8, 5, z, 7, 4)$ or (ii) $(1, x, 7, y, 3, 8, 5, 2)$.
6 cannot be adjacent to 3 so we get:
(i) $(1, 2, 3, 8, 5, 6, 7, 4)$ or (ii) $(1, 6, 7, y, 3, 8, 5, 2)$.
And hence we get two different solutions:
(i) $(1, 2, 3, 8, 5, 6, 7, 4)$, or (ii) $(1, 6, 7, 4, 3, 8, 5, 2)$
after checking that they have no forbidden pairs.

Alternatively, we must alternate odd and even numbers; otherwise we have adjacent numbers summing to an even number larger than 2.
Up to rotation we can have 2 at 12 o'clock.
Then up to reflection, there are only 3 possibilities for where to place the other even numbers:
(i) $(2, x, 4, y, 6, z, 8, t)$, (ii) $(2, x, 4, y, 8, z, 6, t)$, and (iii) $(2, x, 6, y, 4, z, 8, t)$.
The prime sum condition translates to avoiding forbidden odd/even adjacent pairs:

$$1 - 8,\ 3 - 6,\ 5 - 4,\ 7 - 2,\ \text{ and } 7 - 8.$$

Since 7 cannot be adjacent to 2 nor 8, case (ii) is impossible.
For case (i), 7 must be y, and
for case (iii), 7 must be y (between 4 and 6 in both cases).
So we have:
(i) $(2, x, 4, 7, 6, z, 8, t)$, or (iii) $(2, x, 6, 7, 4, z, 8, t)$.
1 cannot be adjacent to 8 so we get:
(i) $(2, 1, 4, 7, 6, z, 8, t)$, or (iii) $(2, 1, 6, 7, 4, z, 8, t)$.
3 cannot be adjacent to 6 so we get:
(i) $(2, 1, 4, 7, 6, z, 8, 3)$, or (iii) $(2, 1, 6, 7, 4, z, 8, t)$.
5 cannot be adjacent to 4 so we get:
(i) $(2, 1, 4, 7, 6, z, 8, 3)$, or (iii) $(2, 1, 6, 7, 4, z, 8, 5)$.
And hence we get two different solutions:
(i) $(2, 1, 4, 7, 6, 5, 8, 3)$, or (iii) $(2, 1, 6, 7, 4, 3, 8, 5)$.
after checking that they have no forbidden pairs.

Alternative argument, with argument in a different order. We must alternate odd and even numbers, otherwise we have adjacent numbers summing to an even number larger than 2.

The prime sum condition translates to avoiding forbidden odd/even adjacent pairs:

$$1-8,\ 3-6,\ 5-4,\ 7-2,\quad \text{and } 7-8.$$

Since 8 cannot be adjacent to either of 1 or 7, 8 must be between 3 and 5.
So up to rotation and reflection, we can assume our circle starts with $(3, 8, 5$.
There are two possibilities for where to place the other odd numbers:
(i) $(3, 8, 5, x, 1, y, 7, z)$, (ii) $(3, 8, 5, x, 7, y, 1, z)$
2 cannot be adjacent to 7 so we get:
(i) $(3, 8, 5, 2, 1, y, 7, z)$, or (ii) $(3, 8, 5, x, 7, y, 1, 2)$.
4 cannot be adjacent to 5 so we get:
(i) $(3, 8, 5, 2, 1, y, 7, z)$, or (ii) $(3, 8, 5, x, 7, 4, 1, 2)$.
6 cannot be adjacent to 3 so we get:
(i) $(3, 8, 5, 2, 1, 6, 7, z)$ or (ii) $(3, 8, 5, x, 7, 4, 1, 2)$.
And hence we get two different solutions:
(i) $(3, 8, 5, 2, 1, 6, 7, 4)$, or (ii) $(3, 8, 5, 6, 7, 4, 1, 2)$
after checking that they have no forbidden pairs.

G. Answer: $(1, 2, 3, 8, 5, 6, 7, 4, 9, 10)$ for instance.
The essential properties for a prime circle for 10 are that the numbers alternate odd and even, and that the following forbidden pairs do not appear:

$$1-8,\ 3-6,\ 5-4,\ 5-10,\ 7-2,\ 7-8,\ 9-6.$$

Examples for $n = 10$, up to rotation, reflection, and swap of 4/10, 3/9 are:

$(1, 4/10, 3/9, 4/10, 7, 6, 5, 8, 3/9, 2)$
$(1, 6, 7, 4/10, 3/9, 4/10, 3/9, 8, 5, 2)$
$(1, 2, 3/9, 4/10, 7, 6, 5, 8, 3/9, 4/10)$
$(1, 2, 5, 6, 7, 4/10, 3/9, 8, 3/9, 4/10)$
$(1, 4/10, 7, 6, 5, 8, 3/9, 2, 3/9, 4/10)$
$(1, 4/10, 7, 4/10, 3/9, 2, 3/9, 8, 5, 6)$
$(1, 6, 7, 4/10, 3/9, 2, 5, 8, 3/9, 4/10)$
$(1, 2, 3/9, 4/10, 7, 4/10, 3/9, 8, 5, 6)$
$(1, 6, 7, 4/10, 3/9, 8, 5, 2, 3/9, 4/10)$
$(1, 4/10, 7, 6, 5, 8, 3/9, 4/10, 3/9, 2)$
$(1, 4/10, 7, 6, 5, 2, 3/9, 8, 3/9, 4/10)$
$(1, 4/10, 7, 4/10, 3/9, 8, 3/9, 2, 5, 6)$

H. The odd/even forbidden pairs are:

$$1-8, 3-6, 5-4, 5-10, 7-2, 7-8, 9-6.$$

This is really the only condition, except for alternating odd/even.
We see that 3 and 9 have the same forbidden set of neighbours, namely $\{6\}$.
So if we swap 3 and 9, we still have a prime circle for 10.
We have to show they are different.

This is clear, since here we fix 8 positions while a rotation fixes none, and a reflection fixes either 0 or 2.

I. For the same reasons we can swap 4 and 10, as they have the same forbidden set of neighbours, namely $\{5\}$.

J. If we start from a prime circle, we can swap 3/9 and/or 4/10, giving us 4 different prime circles.
From one of these four, if we perform these same swaps, we always get the same 4 circles. (In other words we can partition the set of solutions into parts of size 4).
Thus the total number is divisible by 4.
[The exact number is 48 but that's a bit long to compute.]

K. The odd/even forbidden pairs are:

$$1-8,\ 3-6,\ 3-12,\ 5-4,\ 5-10,\ 7-2,$$
$$7-8,\ 9-6,\ 9-12,\ 11-4\ \text{ and }\ 11-10.$$

Hence we can swap 3/9, 5/11, 4/10, and 6/12, as each pair has the same forbidden set of neighbours.
So each solution provides actually $2\cdot 2\cdot 2\cdot 2 = 16$ solutions.
[The number of solutions is $16 \cdot 32$.]
Examples for $n = 12$, up to rotation, reflection, and swap of 4/10, 3/9, 6/12, 5/11.
With $3, 9$ at distance 2:

$$(1, 4/10, 3/9, 2, 3/9, 8, 5/11, 6/12, 5/11, 6/12, 7, 4/10)$$
$$(1, 6/12, 5/11, 8, 3/9, 4/10, 3/9, 2, 5/11, 6/12, 7, 4/10)$$
$$(1, 6/12, 5/11, 2, 5/11, 8, 3/9, 4/10, 3/9, 4/10, 7, 6/12)$$
$$(1, 4/10, 7, 4/10, 3/9, 8, 3/9, 2, 5/11, 6/12, 5/11, 6/12)$$
$$(1, 6/12, 7, 6/12, 5/11, 8, 5/11, 2, 3/9, 4/10, 3/9, 4/10)$$
$$(1, 6/12, 5/11, 8, 5/11, 2, 3/9, 4/10, 3/9, 4/10, 7, 6/12)$$
$$(1, 4/10, 3/9, 4/10, 3/9, 8, 5/11, 2, 5/11, 6/12, 7, 6/12)$$
$$(1, 4/10, 3/9, 8, 3/9, 4/10, 7, 6/12, 5/11, 6/12, 5/11, 2)$$
$$(1, 2, 5/11, 8, 3/9, 4/10, 3/9, 4/10, 7, 6/12, 5/11, 6/12)$$
$$(1, 6/12, 5/11, 6/12, 5/11, 8, 3/9, 2, 3/9, 4/10, 7, 4/10)$$
$$(1, 2, 5/11, 6/12, 5/11, 8, 3/9, 4/10, 3/9, 4/10, 7, 6/12)$$
$$(1, 4/10, 3/9, 8, 3/9, 2, 5/11, 6/12, 5/11, 6/12, 7, 4/10)$$
$$(1, 6/12, 5/11, 8, 5/11, 6/12, 7, 4/10, 3/9, 2, 3/9, 4/10)$$
$$(1, 6/12, 5/11, 8, 5/11, 6/12, 7, 4/10, 3/9, 4/10, 3/9, 2)$$
$$(1, 4/10, 7, 6/12, 5/11, 8, 3/9, 4/10, 3/9, 2, 5/11, 6/12)$$
$$(1, 4/10, 3/9, 8, 3/9, 4/10, 7, 6/12, 5/11, 2, 5/11, 6/12)$$
$$(1, 6/12, 5/11, 8, 3/9, 4/10, 3/9, 4/10, 7, 6/12, 5/11, 2)$$
$$(1, 2, 3/9, 4/10, 3/9, 8, 5/11, 6/12, 5/11, 6/12, 7, 4/10)$$
$$(1, 4/10, 3/9, 4/10, 3/9, 8, 5/11, 6/12, 7, 6/12, 5/11, 2)$$

With $3, 9$ at distance 4:

$$(1, 6/12, 5/11, 8, 3/9, 4/10, 7, 4/10, 3/9, 2, 5/11, 6/12)$$
$$(1, 4/10, 3/9, 8, 5/11, 6/12, 7, 6/12, 5/11, 2, 3/9, 4/10)$$
$$(1, 2, 3/9, 8, 5/11, 6/12, 5/11, 6/12, 7, 4/10, 3/9, 4/10)$$
$$(1, 6/12, 5/11, 6/12, 5/11, 8, 3/9, 4/10, 7, 4/10, 3/9, 2)$$
$$(1, 4/10, 3/9, 8, 5/11, 6/12, 5/11, 6/12, 7, 4/10, 3/9, 2)$$
$$(1, 4/10, 3/9, 2, 5/11, 8, 3/9, 4/10, 7, 6/12, 5/11, 6/12)$$
$$(1, 4/10, 3/9, 8, 5/11, 2, 3/9, 4/10, 7, 6/12, 5/11, 6/12)$$

With $3, 9$ at distance 6:

$$(1, 6/12, 7, 4/10, 3/9, 8, 5/11, 6/12, 5/11, 2, 3/9, 4/10)$$
$$(1, 4/10, 3/9, 8, 5/11, 6/12, 5/11, 2, 3/9, 4/10, 7, 6/12)$$
$$(1, 6/12, 5/11, 2, 3/9, 8, 5/11, 6/12, 7, 4/10, 3/9, 4/10)$$
$$(1, 4/10, 3/9, 8, 5/11, 6/12, 7, 4/10, 3/9, 2, 5/11, 6/12)$$
$$(1, 6/12, 5/11, 8, 3/9, 2, 5/11, 6/12, 7, 4/10, 3/9, 4/10)$$
$$(1, 6/12, 5/11, 8, 3/9, 4/10, 7, 6/12, 5/11, 2, 3/9, 4/10)$$